# Landau Damping of Space-Charge Dominated Fermilab Booster Beam

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# Stability Contour from Métral and Ruggiero

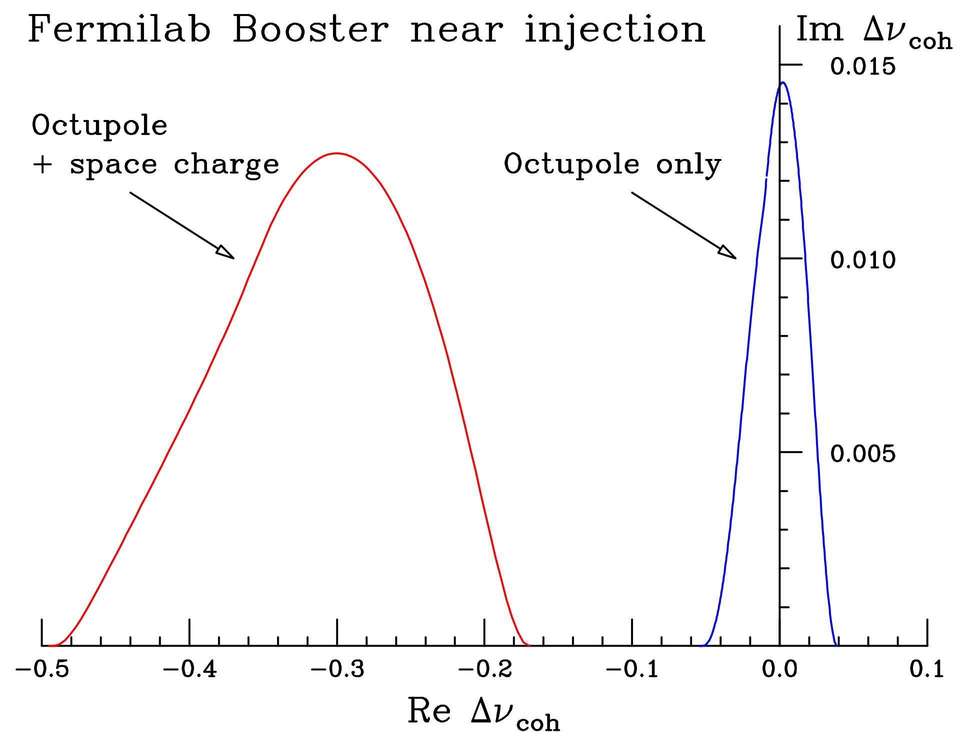
- Stability plot made for Fermilab Booster beam, with trans. dist.

$$f(J_x, J_y) = \frac{12}{J_{\max}^2}\left(1 - \frac{J_x + J_y}{J_{\max}}\right)^2$$

- Needs $\mathcal{R}e\,\Delta\nu_{\rm coh}$ more than $\sim -0.2$ for Landau damping.
  But inductive wall cannot provide it.

- Then, octupole spread is of not much help.

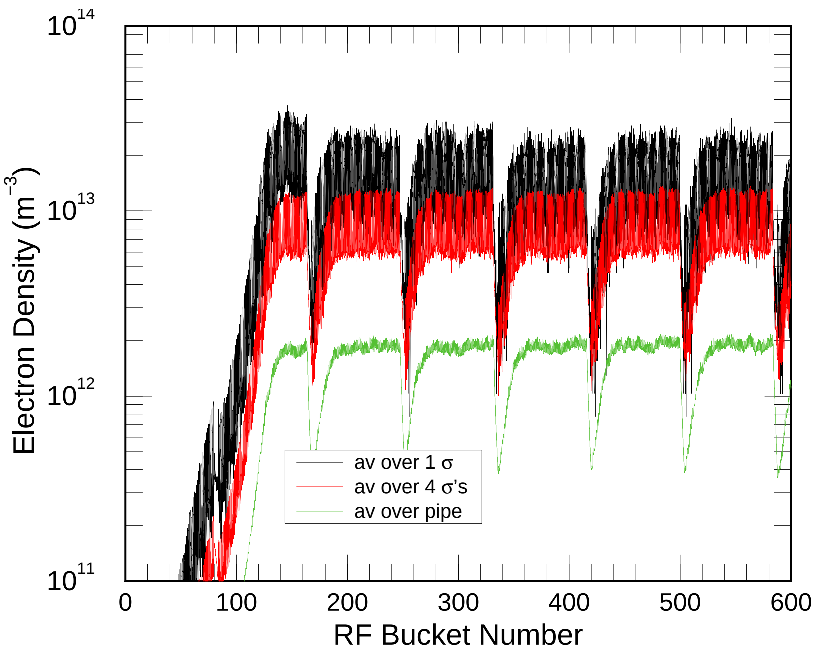
- How can the Booster beam stable?

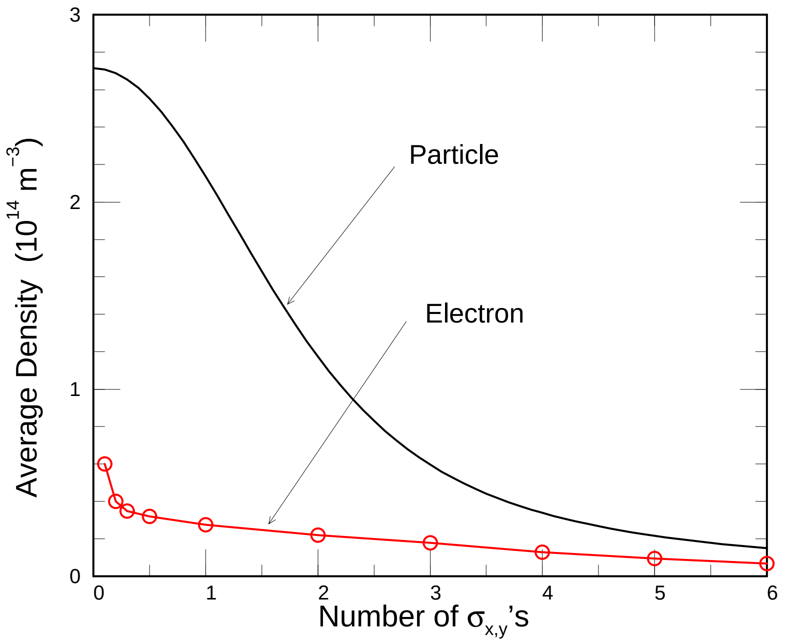



octupole spread $\sim \pm 0.36$.

# Can Electron Cloud Neutralize Space-Charge?

- Electron cloud saturation can occur if SEY$\gtrsim$ 1.6.
  $\sim$100% neutralization on average.
- But electron density inside bunch region not large.
- Only 19% within 2 $\sigma$'s.
  Not large enough to cancel space-charge significantly.
- Also large amount of e-cloud produces severe beam instability.
  But this has not been seen.
  Maybe SEY$<$ 1.6 and no e-cloud accumulation.

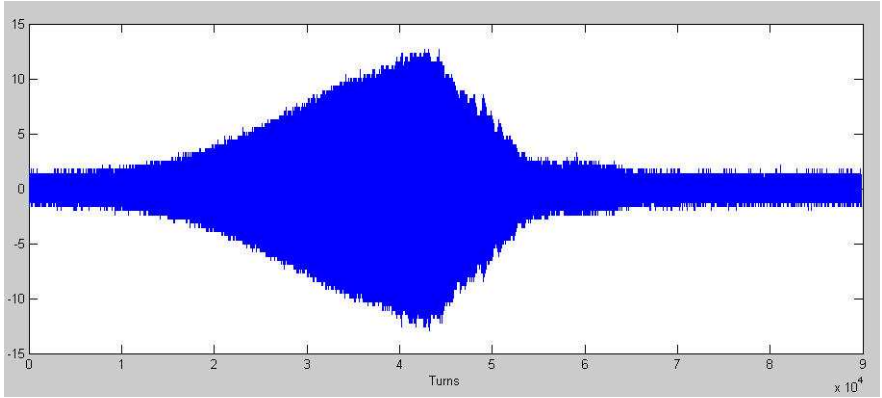


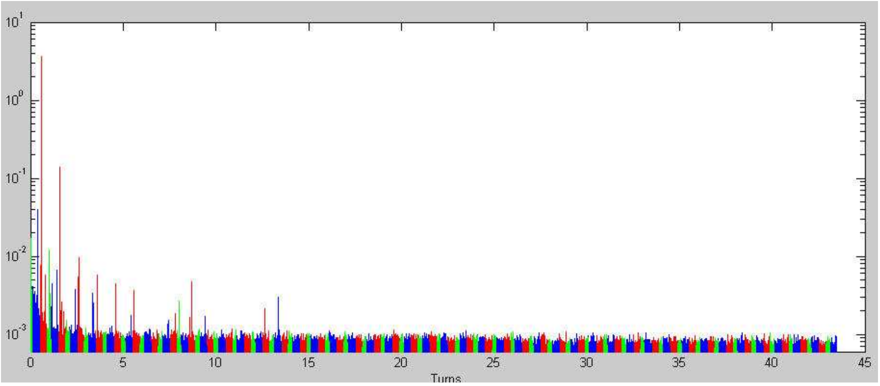



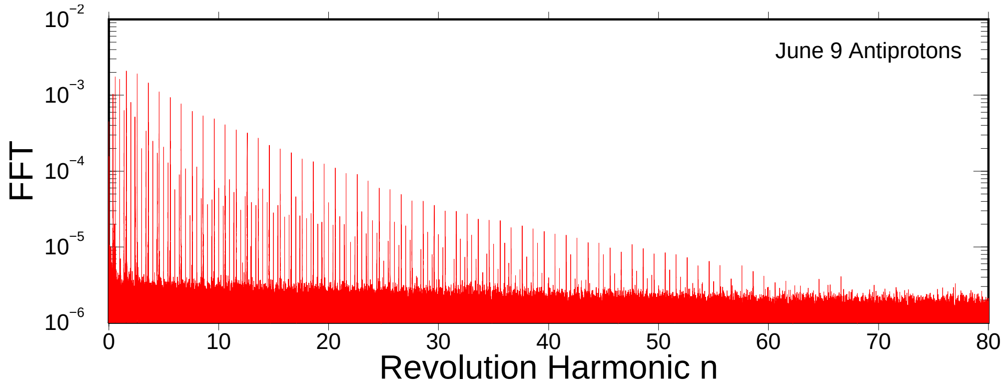
# Property of Coasting Beam

- Métral and Ruggiero stability contours apply to coasting beam only.

- In Fermilab Recycler, we do see such instability driven by wall resistivity. The $\bar{p}$ beam is long and syn. osc. is slow $(T_{\rm syn} \sim 1\ {\rm s})$.



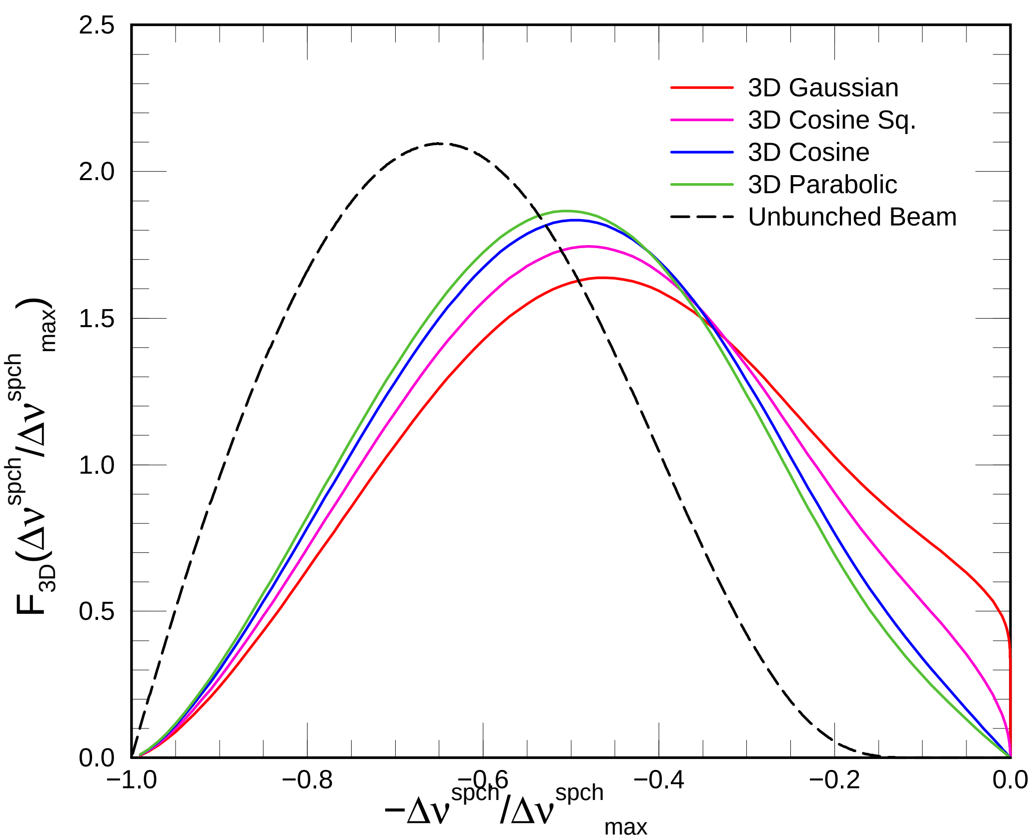




- Instability observed with coasting $p$ beam.

- Need dedicated kicker to restore stability.

# Effects of Bunching

- Coasting beam do not contain particles with small sp ch tune shift.
- Bunch front and tail are of low density.
  Many particles have tiny sp ch tune shift.
- The stability contour of a bunch should provide reasonable Landau damping.



# Dispersion Relation of a Bunch

- Dispersion relation from Métral and Ruggiero:

$$1 = -\int_{-\infty}^{\infty} dJ_x \int_{-\infty}^{\infty} dJ_y \frac{J_y \frac{\partial f(J_x,J_y)}{\partial J_y}\left[\Delta\nu_{\rm coh}^y - \Delta\nu_{\rm inc}^y(J_x,J_y)\right]}{\nu_c - \nu_y(J_x,J_y) - m\nu_s}$$

- Extend to a bunch:

$$1 = -\int_{-\infty}^{\infty} dz \int_{-\infty}^{\infty} dJ_x \int_{-\infty}^{\infty} dJ_y\, g(z) \frac{J_y \frac{\partial f(J_x,J_y)}{\partial J_y}\left[\Delta\nu_{\rm coh}^y - \Delta\nu_{\rm inc}^y(J_x,J_y,z)\right]}{\nu_c - \nu_y(J_x,J_y,z) - m\nu_s}$$

- Incoherent tune shifts $\Delta\nu_y(J_x, J_y)$ include:
  1. Octupole driven tune spread,
  2. Space-charge tune shift.

  Space charge force is fitted to obtain lowest nonlinear term.

- It is nice that the transverse part can be integrated analytically.
  So numerical integration is necessary only over $z$.

# Stability Plot of a Bunch

- Choose a generalized elliptical distribution

$$g(z) = \frac{A_n}{\hat{z}} \left(1 - \frac{z^2}{\hat{z}^2}\right)^n$$

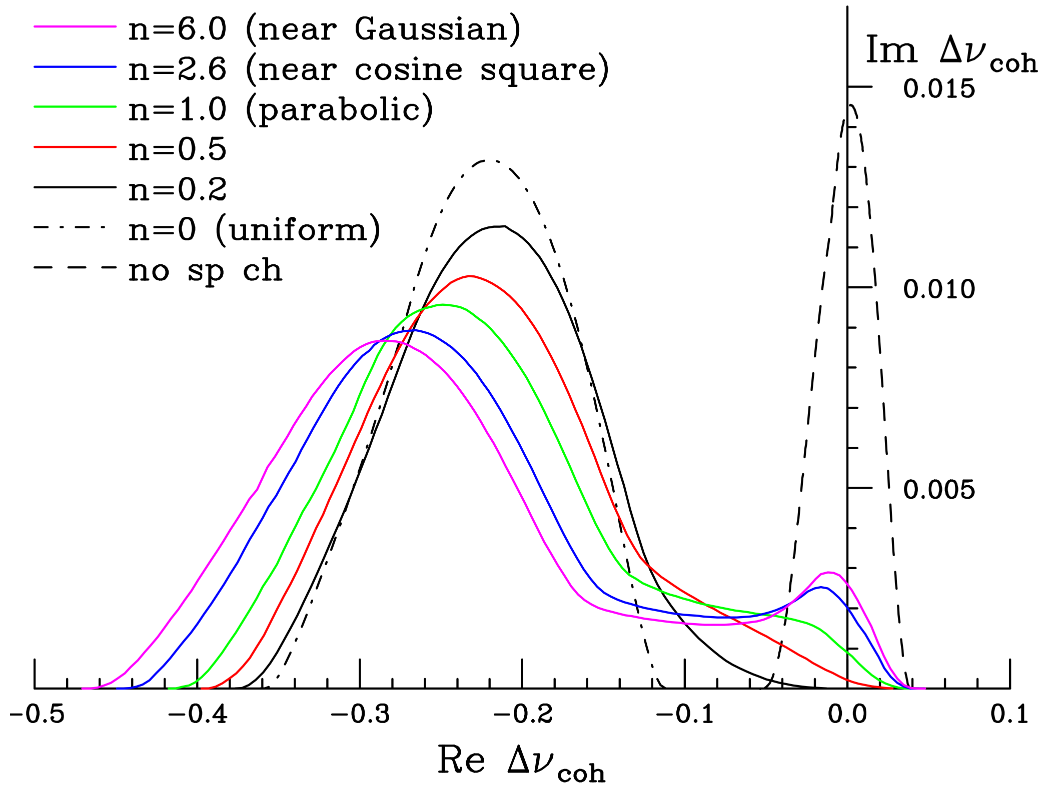
with

$n = 0 \rightarrow$ uniform

$n = 1.0 \rightarrow$ parabolic

$n = 2.6 \rightarrow$ near cosine square

$n = \infty \rightarrow$ Gaussian




- We clearly see a Landau damped region even when $\mathcal{R}e\,\Delta\nu_{\rm coh} = 0$ for $n \gtrsim 1$.

- Stability limit: $\mathcal{I}m\,\Delta\nu_{\rm coh} \approx 0.002.$ when octupole spread $\sim 0.36$. Without Landau damping, this amounts to growth time $\tau = 0.17$ ms.

- Stability limit drops very fast as octupole spread decreases, e.g., to 0.0002 when octupole current is reduced to 30%.

## Coherent Tune Shift vs Impedance

- Solving Sacherer's integral equation and ignoring mode coupling, we get

$$\left[\Delta\nu_{\rm coh}\right]_{\mu mk} = -i\frac{r_p N_b}{2\pi\gamma\nu_\beta Z_0} Z_1^\perp\Big|_{\rm eff}^{\mu mk}$$

$$Z_1^\perp\Big|_{\rm eff}^{\mu mk} = M\sum_n Z_1^\perp(\omega_n)\left|\tilde{\lambda}_{mk}(\omega_n-\omega_\xi)\right|^2,$$

with $\omega_n=(Mn+\mu+\nu_\beta+m\nu_s)\omega_0$

$\tilde{\lambda}_{mk}(\omega_n)$ is spectrum of mode $mk$

and Hermite modes will be used in computation.

- Notice that one value of $\Delta\nu_{\rm coh}$ corresponds to a combination of many values of $Z_1^\perp$.

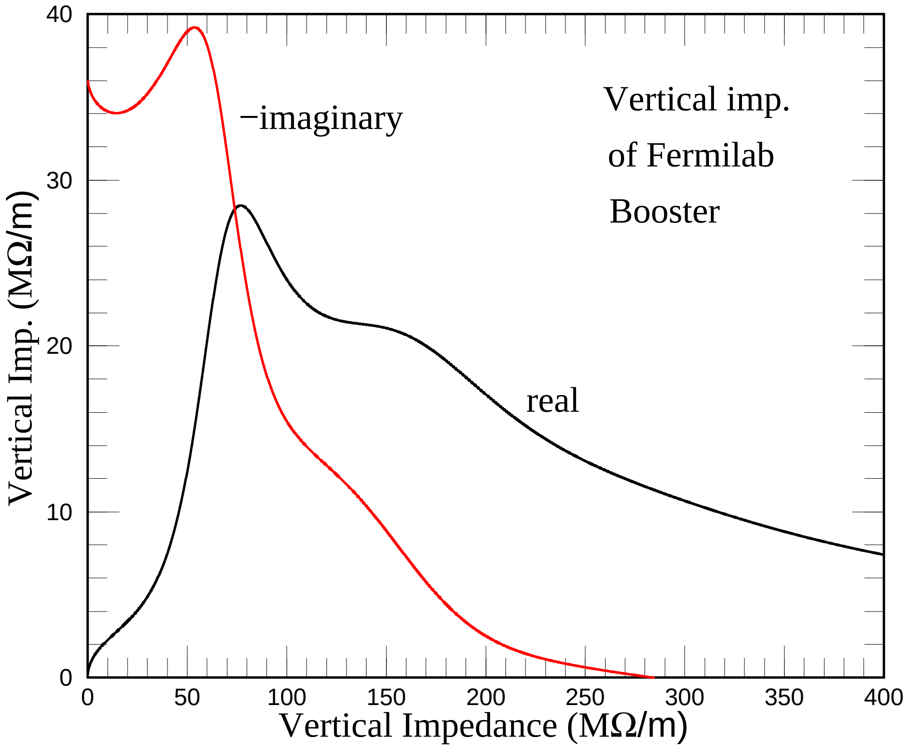
- This explains why $\Delta\nu_{\rm coh}$ is used in stability plot rather than $Z_1^\perp$.

# Application to Fermilab Booster

## *Transverse Impedance*

- Booster consists of 60% unshielded magnets and 40% beam pipes.

- Imp. of laminated magnets has been computed.
  Real part bends back to zero around 80 MHz.
  It looks like a broad-band and does not contribute much to coupled-bunch instabilities.




- Beam pipes: $Z_1^\perp\Big|_{\rm pipe} = \big[\,\mathrm{sgn}(\omega) - i\,\big]\dfrac{0.199}{\sqrt{|\omega/\omega_0|}}\ \mathrm{M}\Omega/\mathrm{m}$,

  which is too small to be shown.
  Real part bends back to zero around 100 Hz, max. $\sim 12.9\ \mathrm{M}\Omega/\mathrm{m}$.
  Contribute mostly to coupled-bunch instabilities.

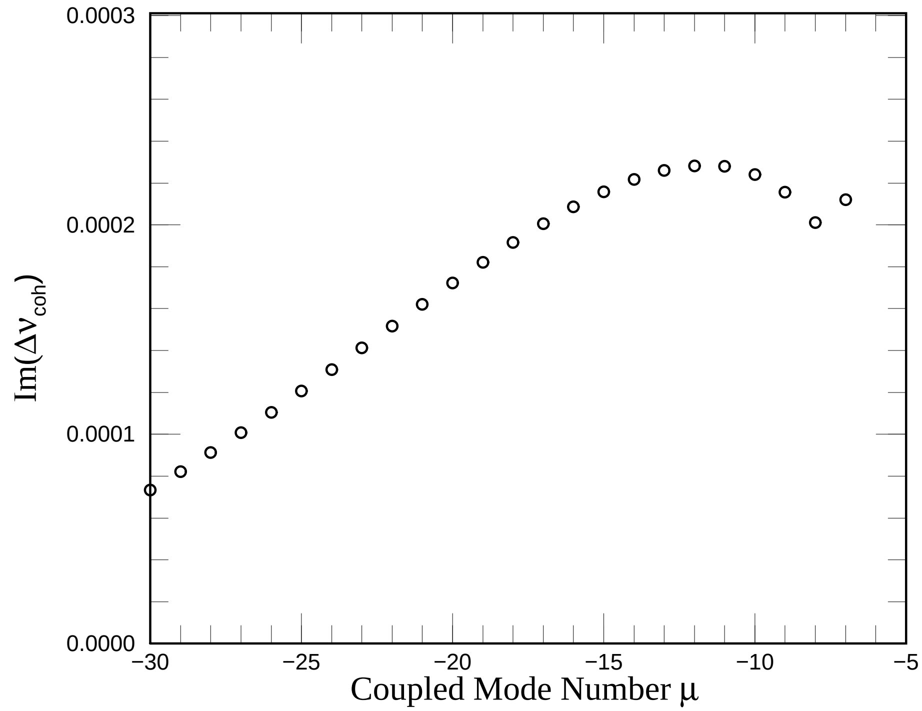
# Coupled-Bunch Instabilities

- 84 bunches at $6 \times 10^{10}$ each.
  Vertical tune $\nu_y \sim 6.8$.

- Most unstable coupled mode is
  $\mu = -7$.
  But lamination contribution shifts it
  to $\mu = -12$.
  For the $m = 0$, $k = 0$ mode,

$$\Delta\nu_{\rm coh} = -0.26 + i0.00023$$

  which is inside stable region.

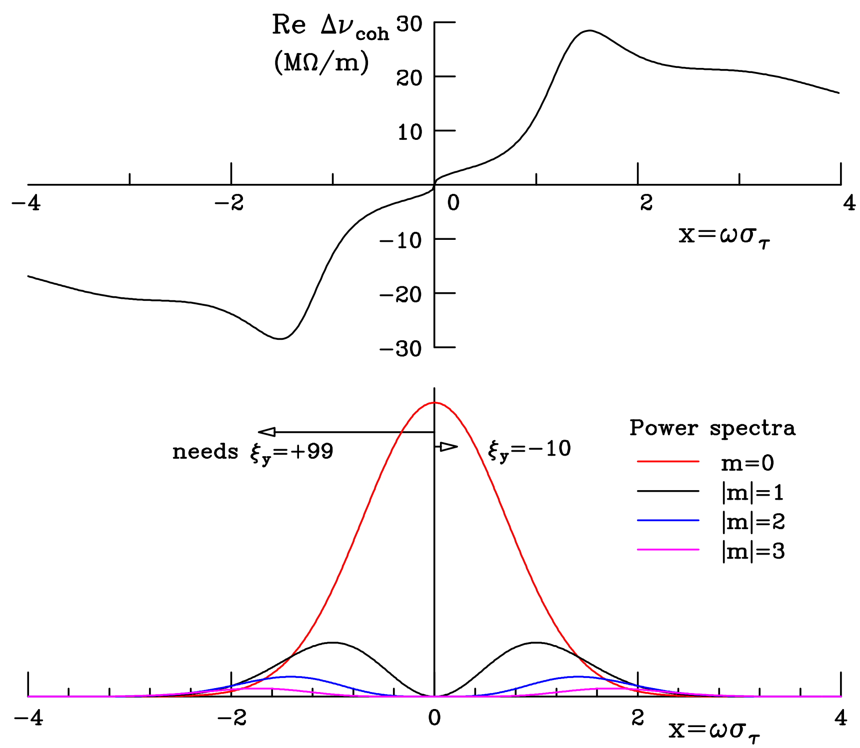



- Without Landau damping, this corresponds to growth rate $\tau = 1.48$ ms.

- Chromaticity will not lead to modes with much higher $\mathcal{I}m\,\Delta\nu_{\rm coh}$.

# Single-Bunch Head-Tail Instabilities

- Only broad-band contributes.
  $\xi_y \neq 0$ is required for instability.
- Most unstable: mode $m = 0$ moved to broad-band imp.
  Requires $\xi_y \sim 99$, too large.
  Booster operates at most between $\xi_y \sim \pm 10$, since $\nu_y \sim 6.8$.
- At $\xi_y = +10$,
  $\Delta\nu_{\rm coh} = -0.025 + i0.0022$,
  on edge of stability region.



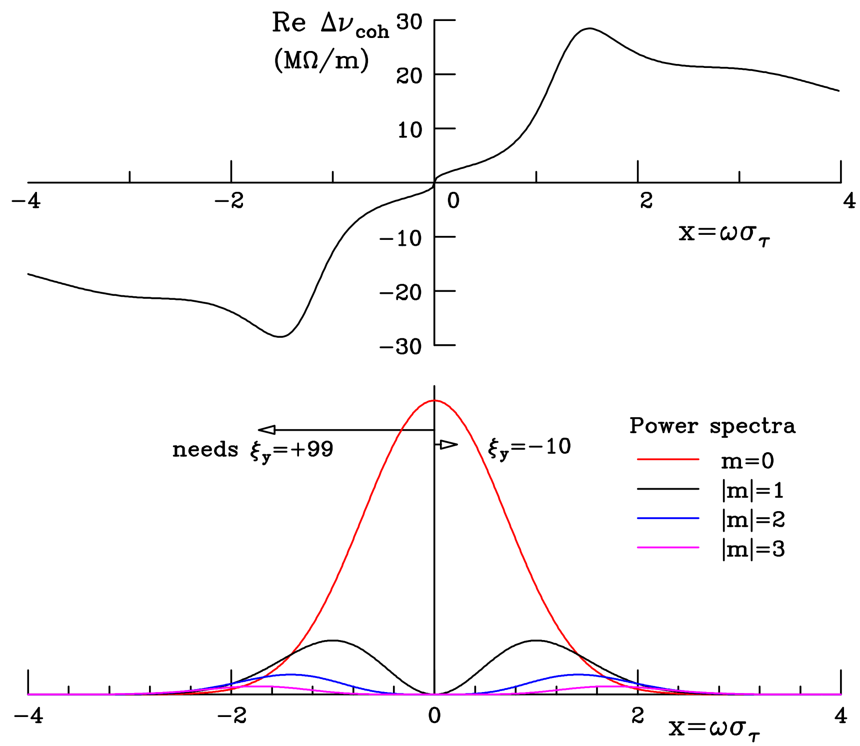

- Booster usually operates at $\xi_y < 0$ near injection (below transition) so that $m = 0$ is stable, modes $|m| > 3$ will be unstable.
- To make mode $|m| = 4$ most unstable,
  needs $\xi_y \sim -12$ to move it to broad-band peak.
  Then, $\mathcal{I}m\,\Delta\nu_{\rm coh} = 0.000052$, very small, certainly inside Landau damped area.

- This implies that all head-tail modes stable near injection.

- Reasons:

  1. slip factor rather large, $\eta \sim -0.4154$ near injection, making $\xi_y$ inefficient to move power spectra.

  2. peak of Hermite modes decrease as $e^{-|m|}$. Higher order modes will not contribute to large growth.




- This makes Booster stable near injection even if $\xi_y$ has the wrong sign if it is not too big.

- This will not be true near transition.
  Power spectra can be shifted by large amount even when $|\xi|$ is small.
  But space-charge decreases by very much, so that octupole-spread damping becomes more efficient.

# Conclusion

- We derive dispersion relation for a bunch with strong space charge plus octupole tune spread, and map out the stability contour.

- Stability plot of $\Delta\nu_{\rm coh}$ shows finite Landau damping region even at $\mathcal{R}e\,\Delta\nu_{\rm coh}=0$.

- Applied to Fermilab Booster near injection with octupole tune spread of $\pm 0.36$, this implies Landau damping when $\mathcal{I}m\,\Delta\nu_{\rm coh}\lesssim 0.002$.
  Without Landau damping, this corresponds to growth time $\tau\gtrsim 0.17$ ms.

- Both single-bunch and coupled-bunch instabilities should be Landau damped with reasonable chromaticity $|\xi_y|\lesssim 10$ (near injection).

- We now understand how octupole tune spread can provide Landau damping to a bunch with strong space charge, but not necessary to a coasting beam.