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Equilibrium analysis of current ramp-up plasma in the LATE device

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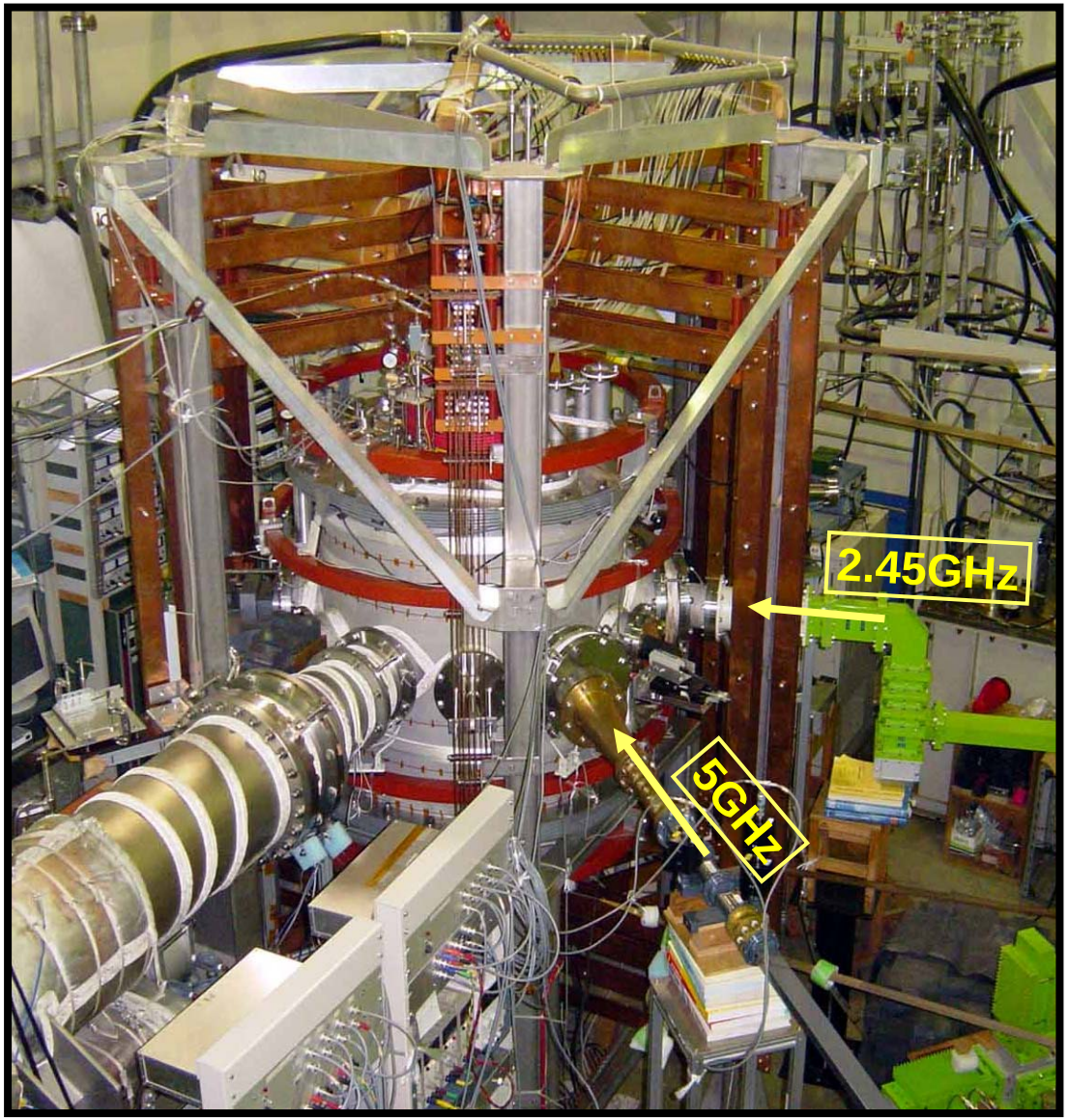
Low Aspect ratio Torus Experiment (LATE) is exploring non-solenoidal start-up by ECH/ECCD



Device Parameters:

Vacuum vessel :
diameter = height = 1m
Center post : diam. = 11.4 cm
Toroidal coils :
Bt = 0.48 kG (R=25cm), 10 s
Bt = 1.15 kG (R=25cm), 0.3 s
Vertical coils: 3 sets
Vertical position control

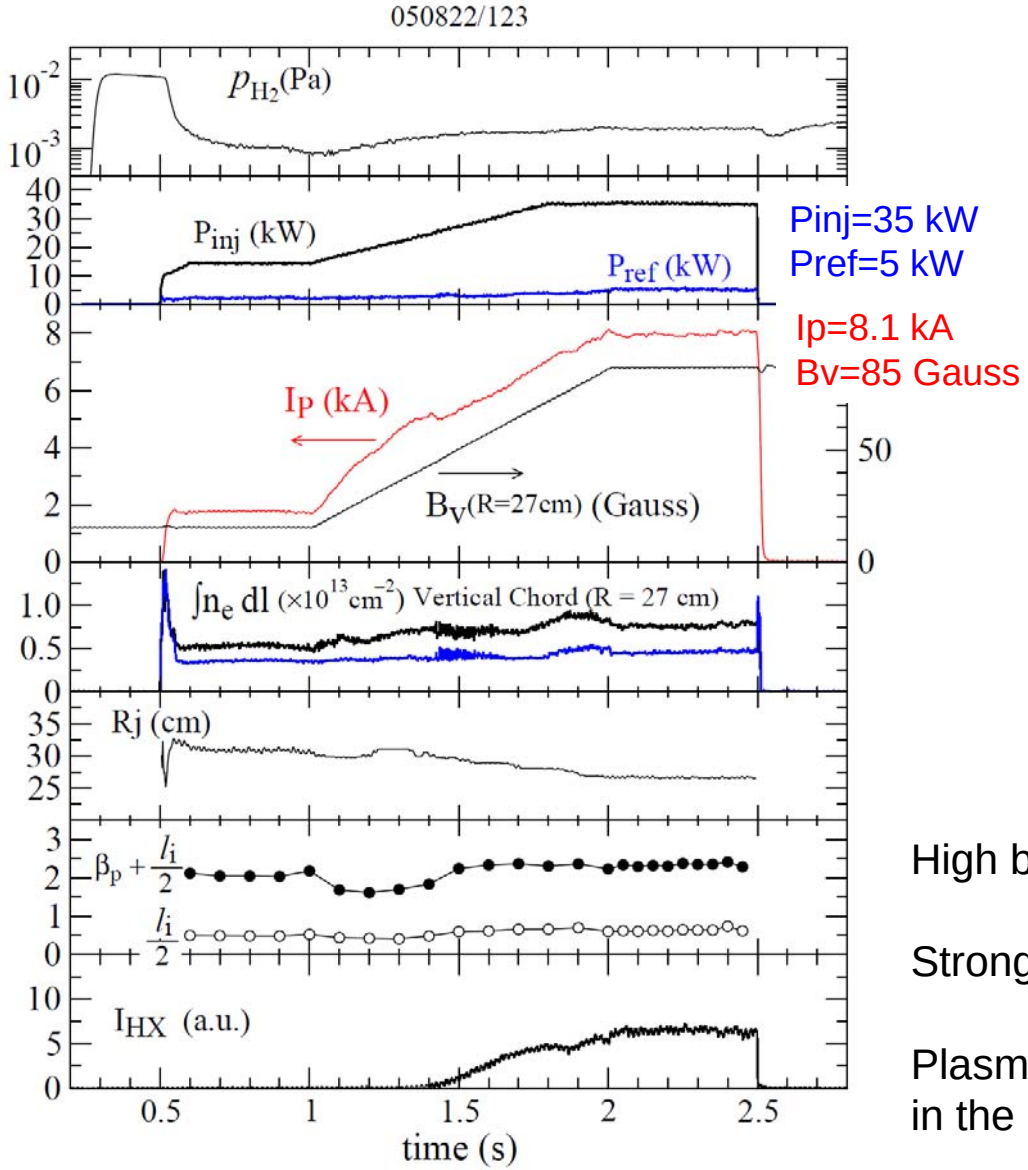
Microwaves:

2.45 GHz 5kW CW x 2,
20kW 2s x 2
5.0 GHz 200kW 0.07s

Diagnostics:

Magnetics (17 Flus loops),
70GHz interferometer (3 chords),
SX cameras (4-poloidal, 1-toroidal), X-ray PHA (CdTe),
Fast visible camera,
Langmuir probes, Spectrometer

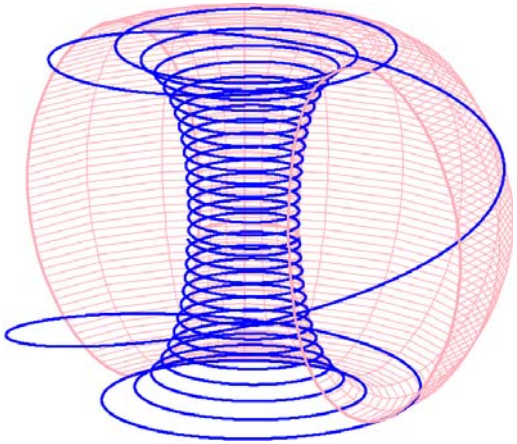
2.45 GHz, 2 seconds, 8.1 kA Discharge



Pinj=35 kW
Pref=5 kW

Ip=8.1 kA
Bv=85 Gauss

Field line on the last closed flux surface (Ip=8.1 kA)

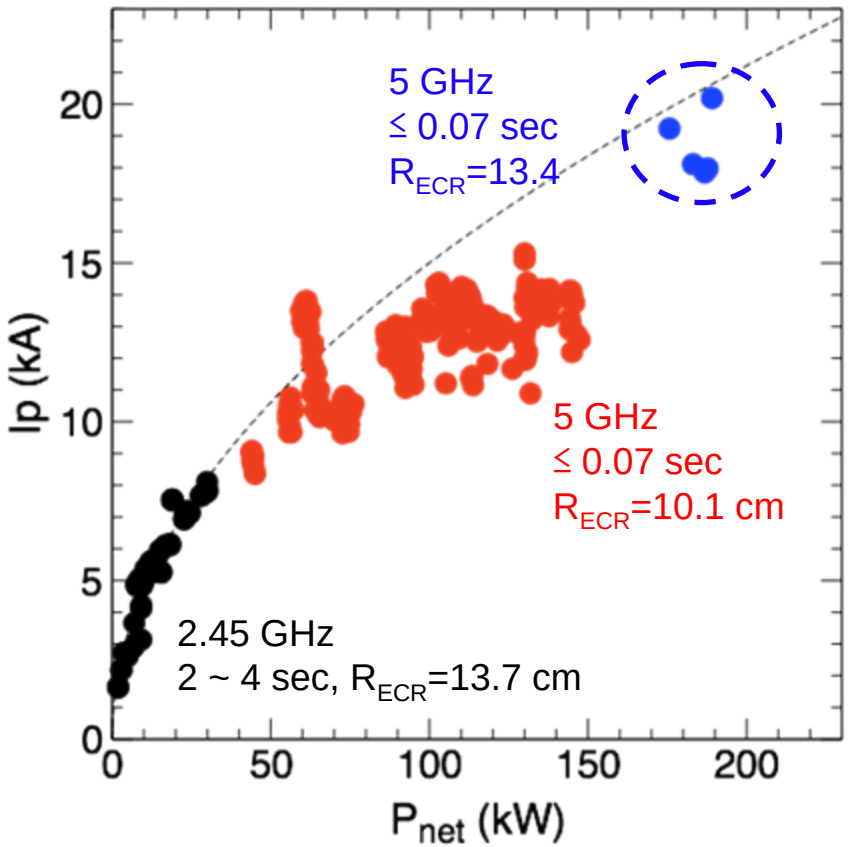


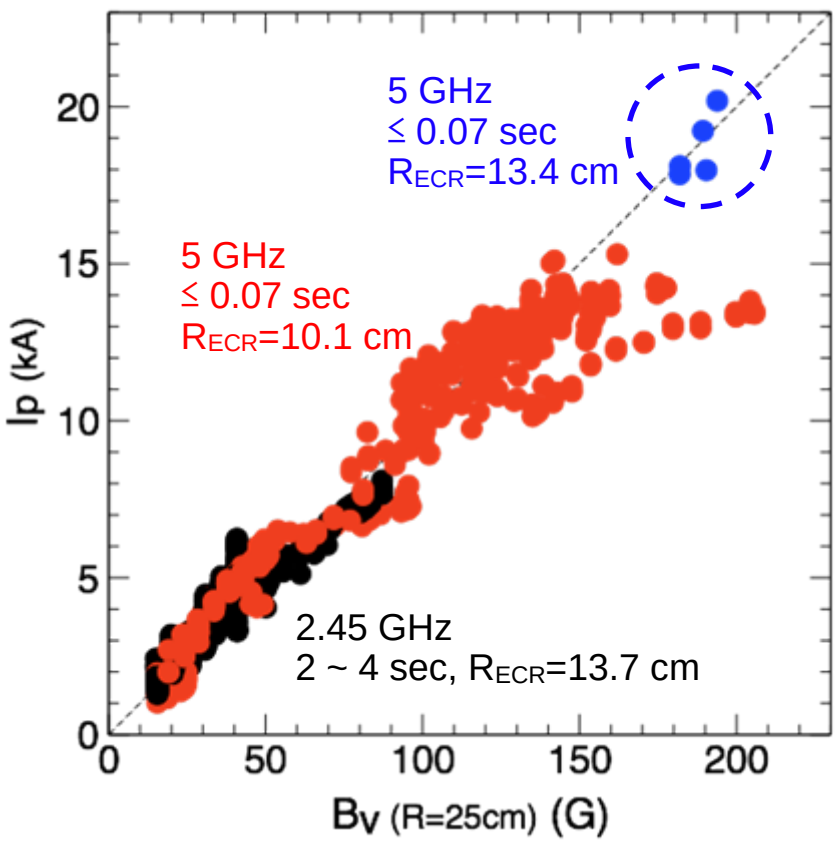
High beta-poloidal β p ~ 1.5

Strong Hard X-ray emission

Plasma Current carried by energetic electrons in the range of ~100 keV

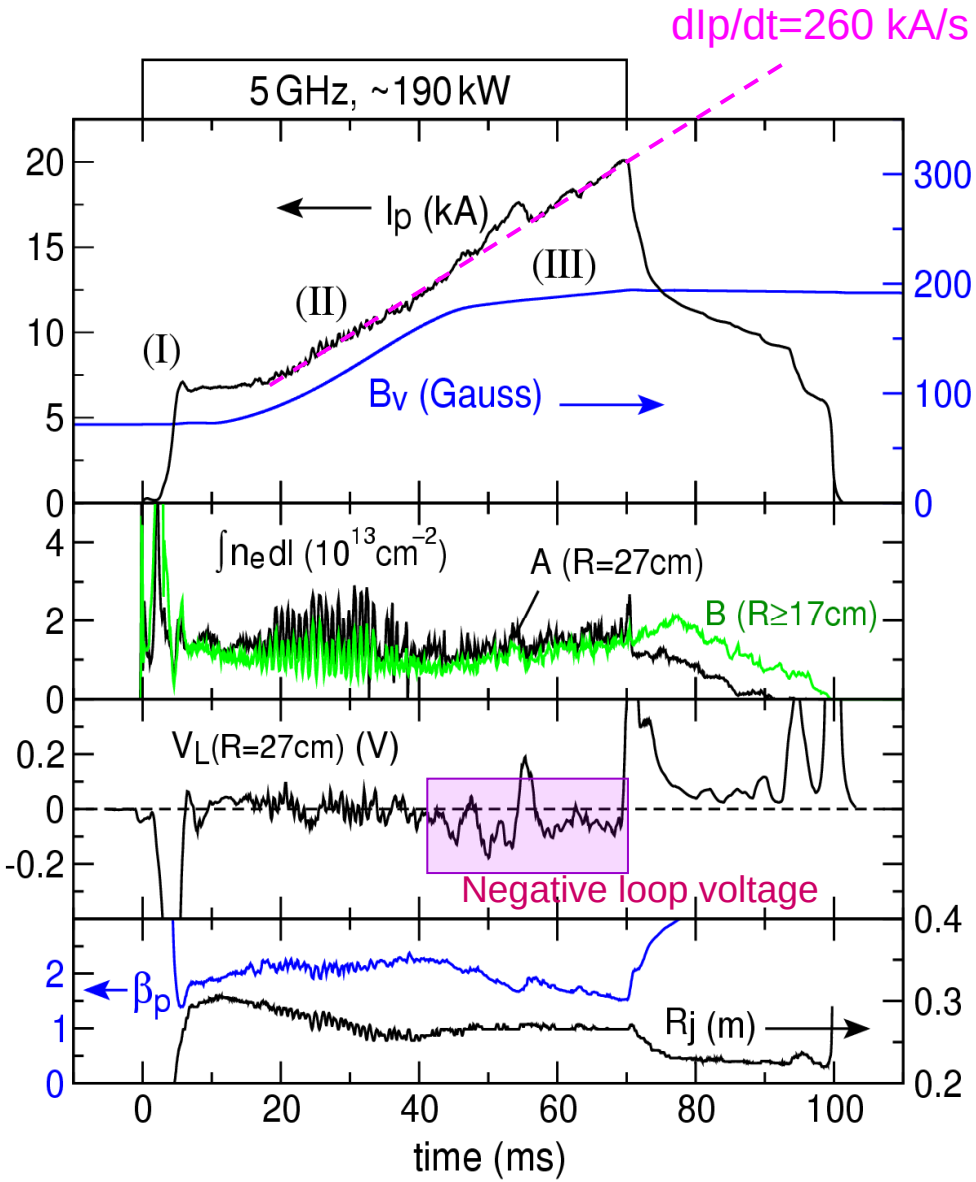
Ip increases with Prf and Bv





Pnet = Pinj - Pref

Current ramps rapidly up to 20kA by 5 GHz EC power and Bv control



Discharge evolves in three stages

Ⅰ Closed surface is formed under steady Bv

Ⅱ Ip ramps up with a ramp of Bv

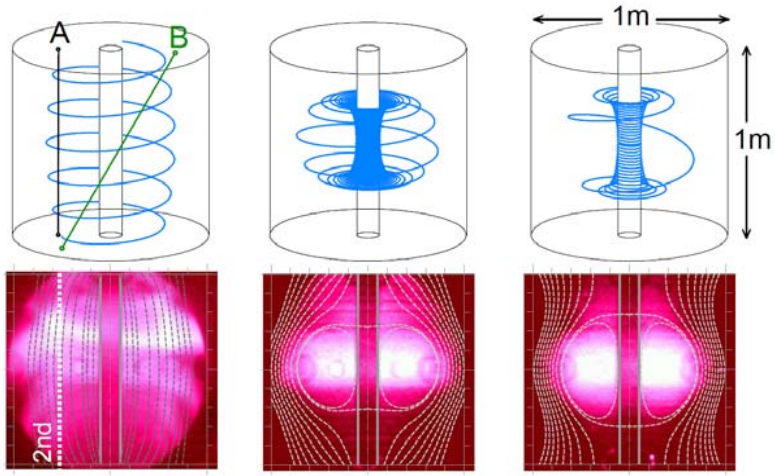
Ⅲ Bv-ramp is turned very slow. Ip still ramps up.

⇒ Negative loop voltage VL~ - 0.1 V

β p∼1.5 due to tail

(β p-thermal <0.05) when Ip=20kA

⇒ Fast electron tail energy ~ 100 keV



Ip=0 Ip=7 kA Ip=20 kA

Fast Electron Tail carries Plasma Current

(1) Average electron drift velocity is comparable to bulk electron thermal velocity, which excludes possibility of bootstrap current.

$$\langle \mathrm{v}_{//} \rangle \equiv \frac{I_\mathrm{p}}{\mathrm{enS}} \approx 3\times 10^6\, m/s \quad for\ \ I_\mathrm{p} = 20kA,\ \ n \approx 4\times 10^{17}\, m^{-3},\ \ S \approx 0.1\, m^2$$

$$\mathrm{v}_\mathrm{th} \equiv \sqrt{T_e/m} = 4.2\times 10^6\, m/s \ \ even\ \ for\ \ T_e = 100\, eV$$

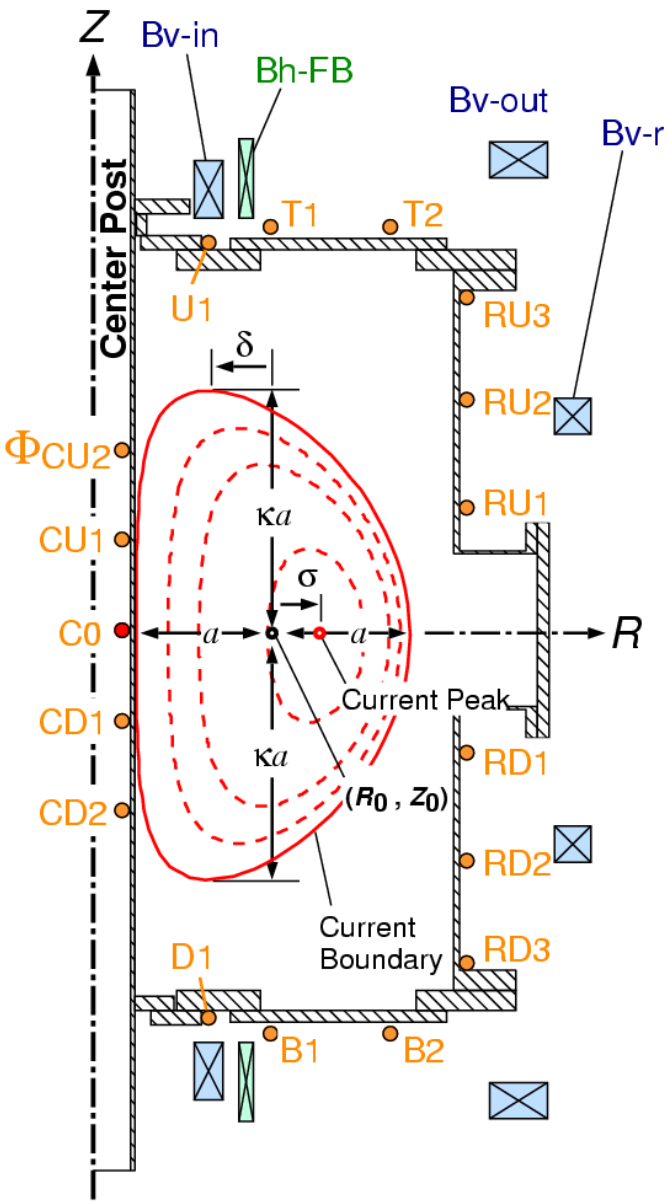
(2) Average electron energy <W> is very large.

$$\langle W \rangle \equiv \frac{\mu_0 I_\mathrm{p}^2 \beta_\mathrm{p}}{8\pi nS} \approx 4.7\, keV$$

$$for\ \ I_\mathrm{p} = 20\, kA,\ \ \beta_\mathrm{p} = 1.5,\ \ \ n = 4\times 10^{17}\, m^{-3},\ \ S = 0.1\, m^2$$

$$, while\ \beta_p(thermal) \approx 0.05\ at\ most.$$

Poloidal Field Coils and Flux Loops



Poloidal Field Coils:

Bv-in, Bv-out and Bv-r coils for vertical field

Bh-FB coil for vertical position control

Flux Loops:

CD2, CD1, C0, CU1, CU2 : Inboard

U1, T1, T2 : Top

D1, B1, B2 : Bottom

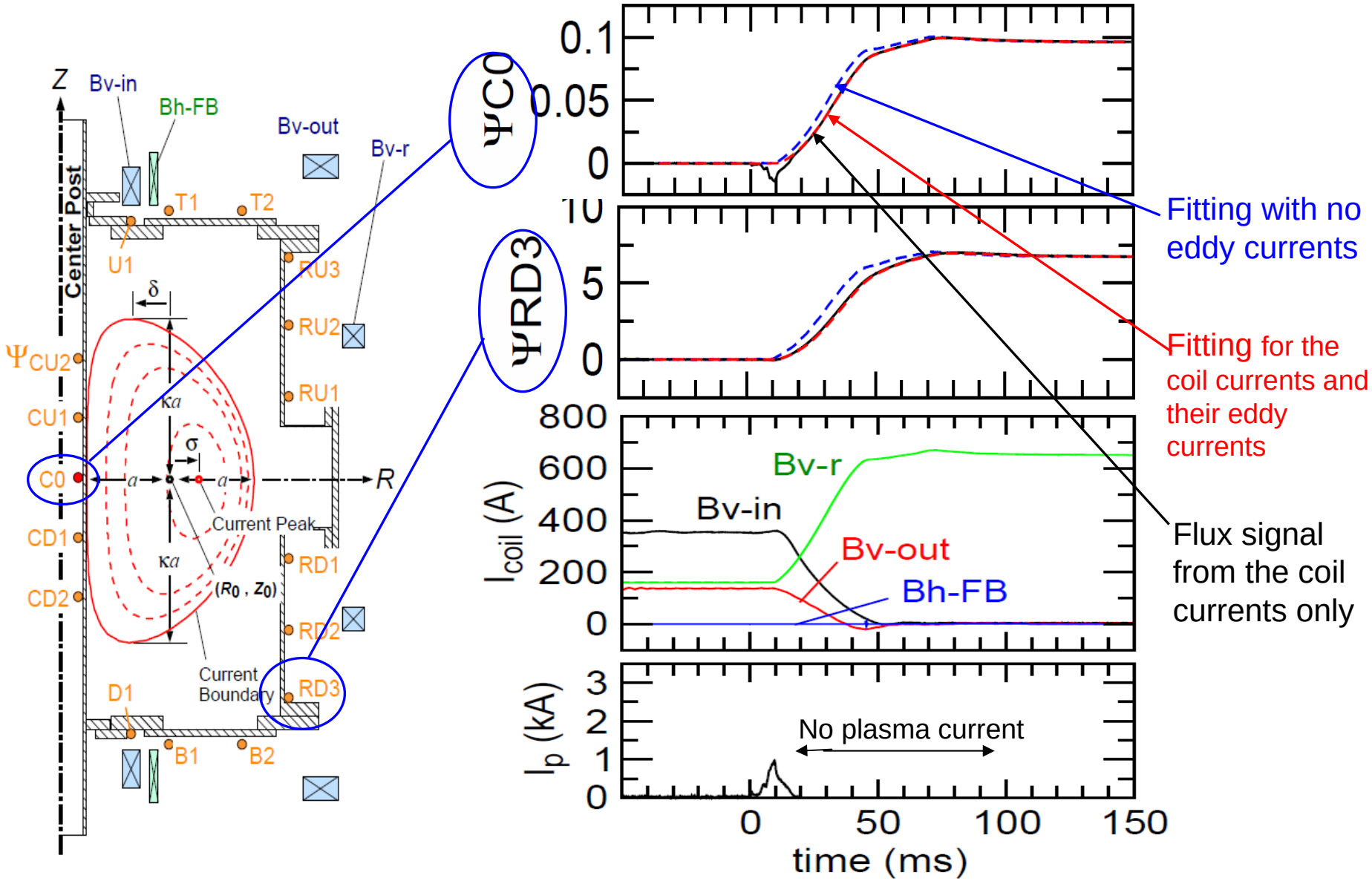
RD3, RD2, RD1, RU1, RU2, RU3 :Outboard

1. Power supplies are all transistor-regulated with no switching noise.
2. No flux swing from CS (No CS)
3. Flux loops are insensitive to local eddy currents induced in local structures such as viewing ports and flanges. (Estimation of these local eddy currents is quite difficult.)

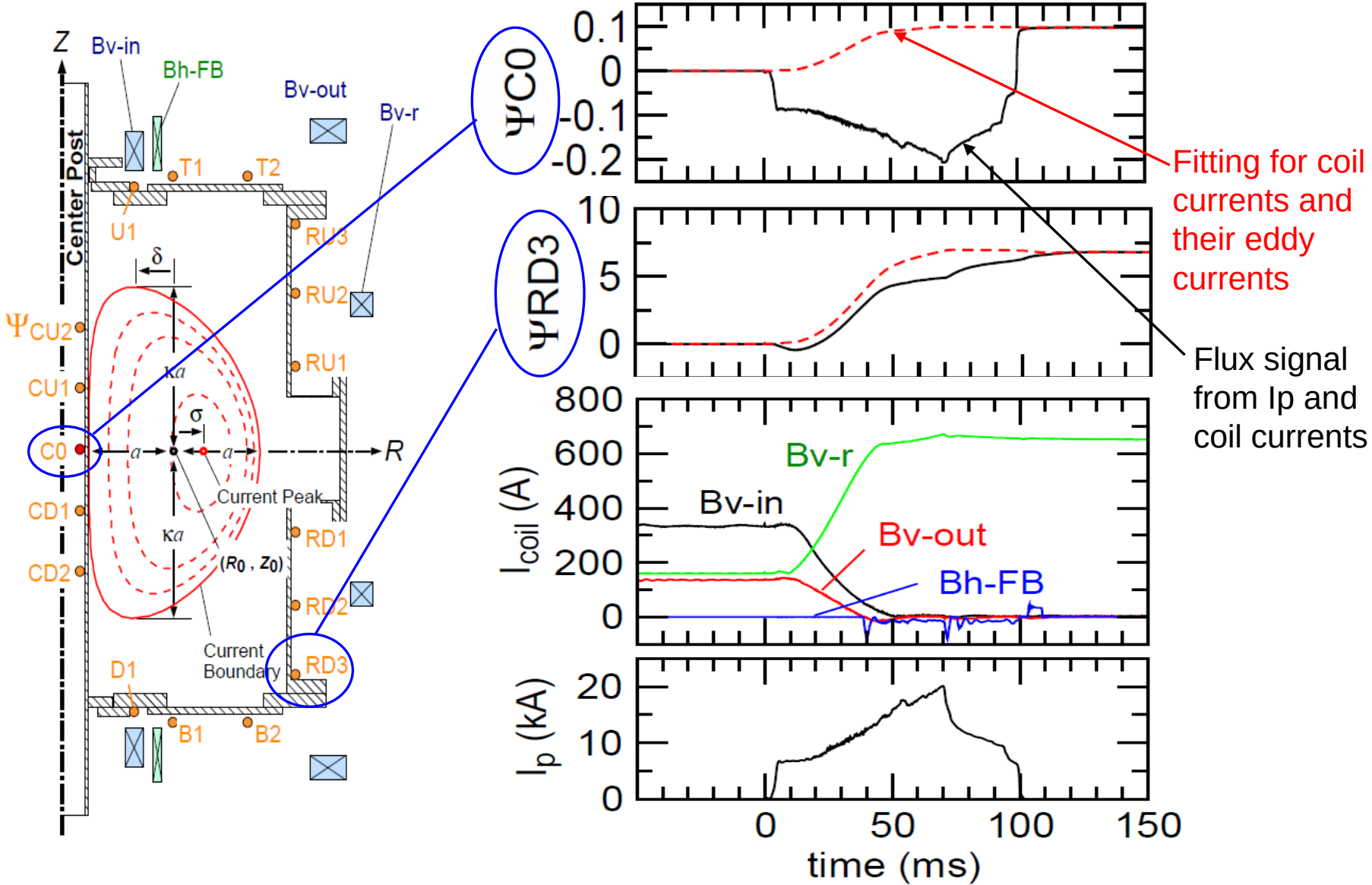
Procedure of magnetic analysis to obtain current profile

1. First, we estimate the contributions to flux loops from poroidal field coil currents and their eddy currents. The eddy currents are estimated by modeling the vacuum vessel by 24 segments of conducting cylinder with uniform current.
2. Then, we subtract above contributions from the flux signals
3. Finally, we fit the contributions from model plasma current profiles to the subtracted signals

Estimation of contributions to flux loops from poroidal field coil currents and the eddy currents.

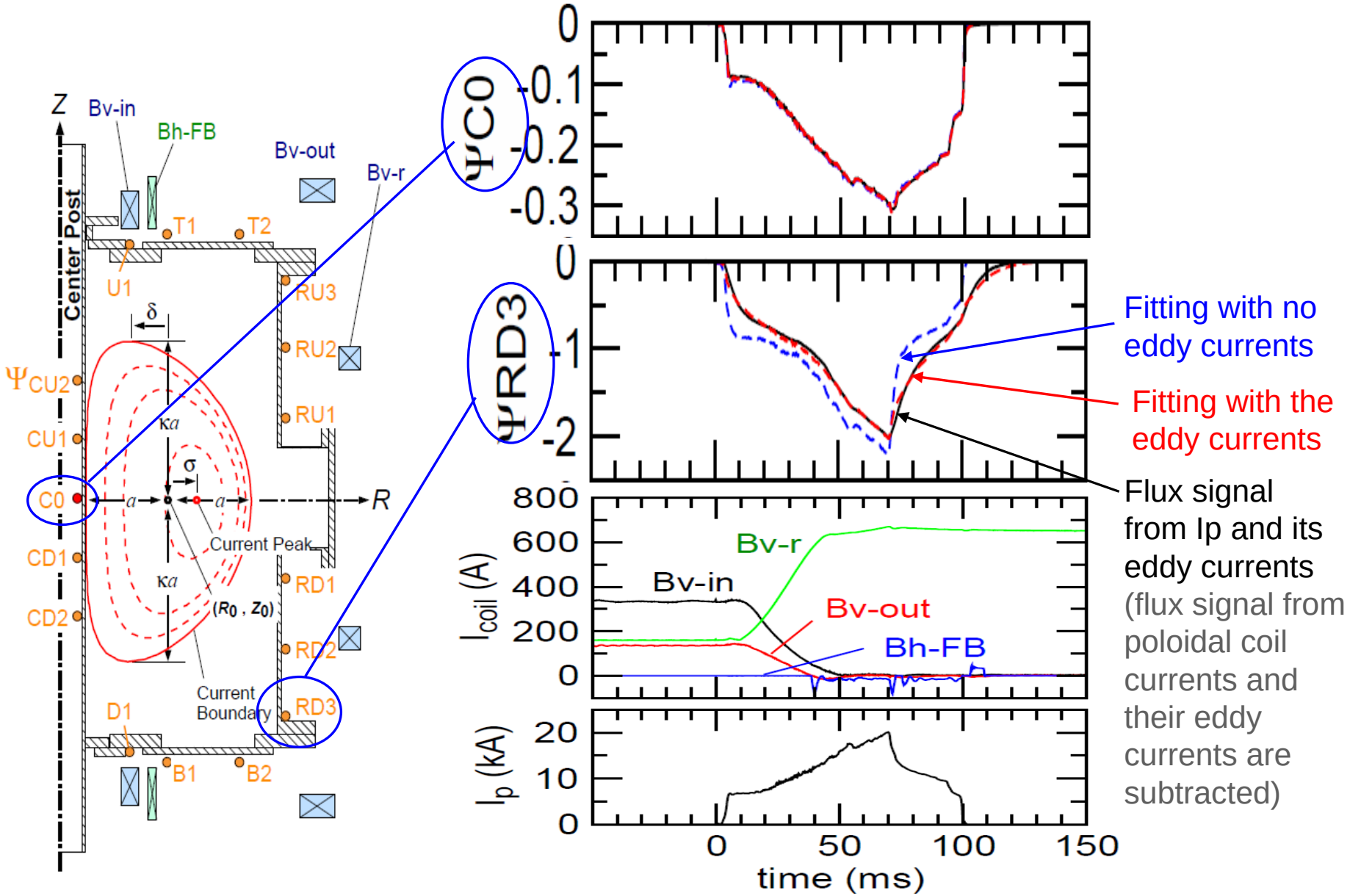


Subtraction of contributions of the coil and eddy currents from flux signals



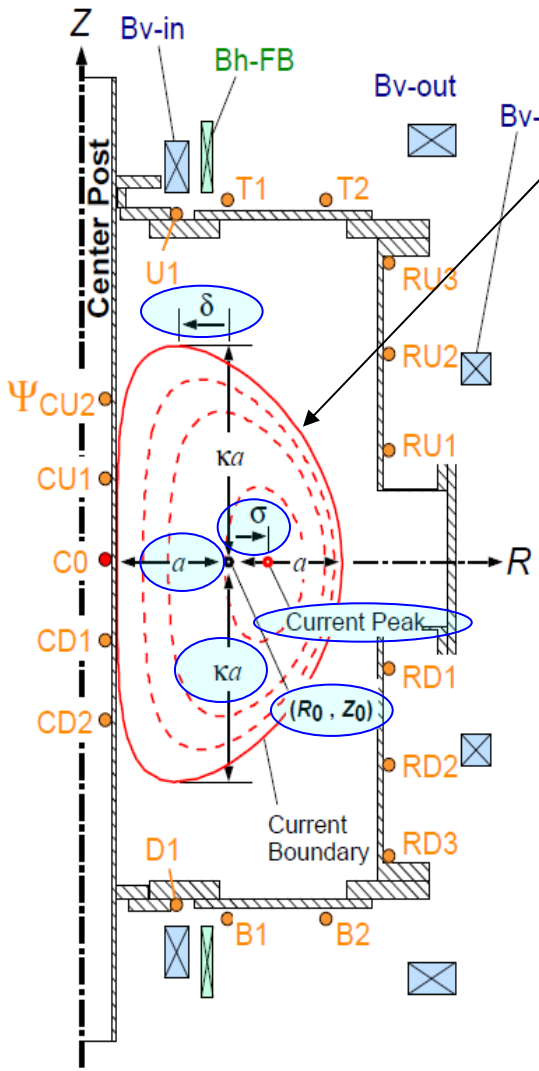
Fitting of model plasma current profile to the subtracted signals

Differences between flux signals and fitted ones are less than 1 % in average.



Accurate fitting allows us to estimate current profile itself in addition to $\beta_P + \ell_i/2$.

Thus $S\langle P\rangle = \mu_0 I_P^2 \beta_p / 8\pi$ can be deduced, *where S is the plasma cross section.*



Current profile model

$$j = j_0 \left\{ 1 - \left(\frac{a_x}{a} \right)^2 \right\}^{\nu}$$

$$a_x = f\,(R_0, Z_0, a, \kappa, \delta, \sigma)$$

Fitting parameters are:

Profile center (R_0, Z_0)

Radius of current boundary (a)

Ellipticity (κ)

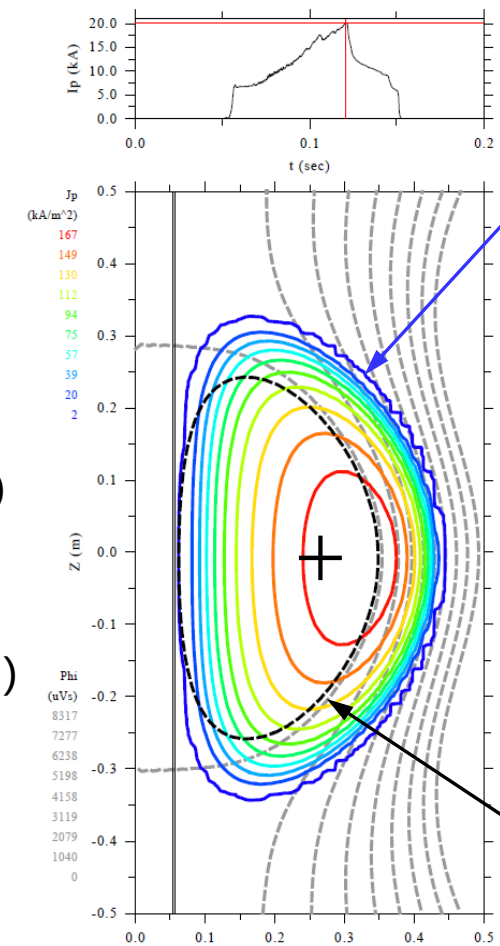
Triangularity (δ)

Radial shift of current peak (σ)

Peaking index (ν)

Peak current density (j_0)

Result for 20 kA plasma



Radial force balance

Shafranov formula for large aspect ratio plasmas with circular cross section:

$$B_V = \frac{\mu_0 I_P}{4\pi R_0}\left\{\ln\left(\frac{8R_0}{a}\right) + \frac{\ell_i}{2} - \frac{3}{2} + \beta_P\right\}, \quad \beta_P = \frac{8\pi S\langle P\rangle}{\mu_0 I_P{}^2}, \quad S = \pi a^2$$

This formula may be generalized;

$$B_V = \frac{\mu_0 I_P}{4\pi R_0}\left\{G\left(\frac{R_0}{a}, \kappa, \ell_i, \ldots\right) + \beta_P\right\}, \quad \beta_P = \frac{8\pi S\langle P\rangle}{\mu_0 I_P{}^2}$$

$$\Rightarrow \quad R_0 I_P B_V = \frac{\mu_0}{4\pi} G\left(\frac{R_0}{a}, \kappa, \ell_i, \ldots\right) I_P{}^2 + 2S\langle P\rangle$$

Current hoop expanding term

Pressure ballooning term

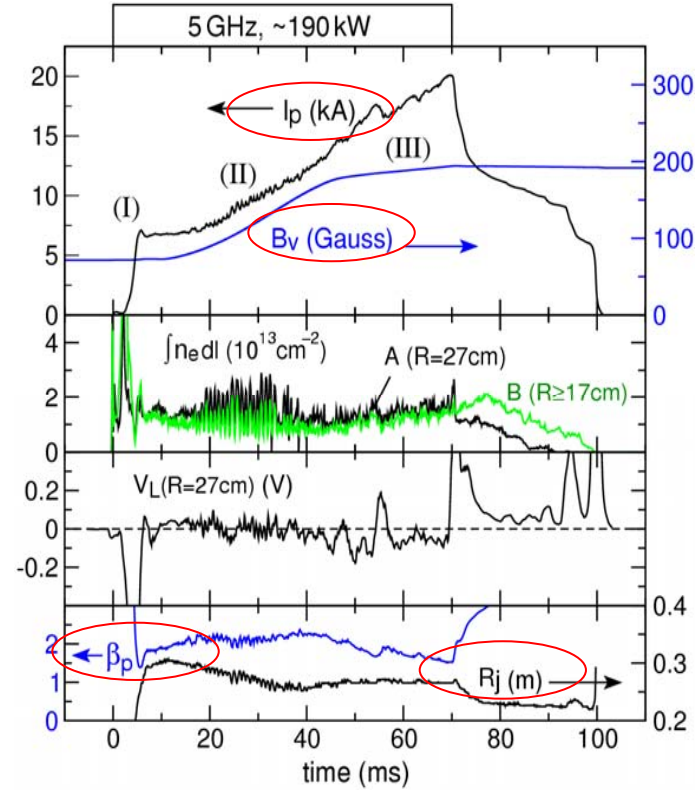
The R_0IpBv force balances two expansion forces.
G is a parameter that reflects geometrical factors of current profile.

The 20 kA discharge is compared with the formula for various values of G

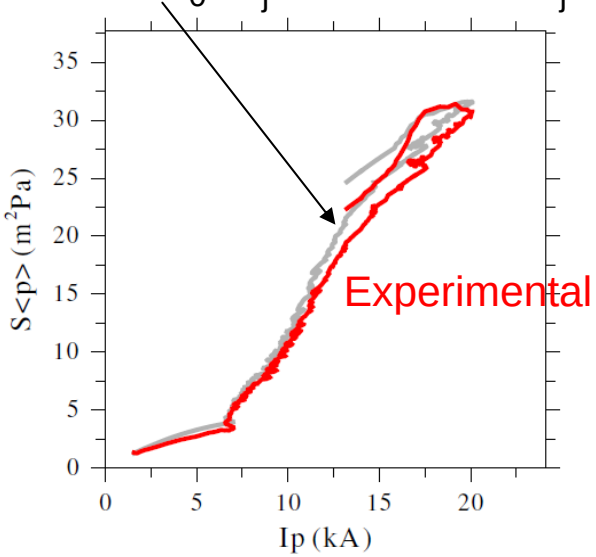
$R_0 B_V$, I_P, *and* $S\langle P\rangle = \frac{\mu_0}{8\pi} I_P^2 \beta_P$ *are related through* G *as,*

$$S\langle P\rangle = \frac{1}{2}\left\{ R_0 B_V I_P + \frac{\mu_0}{4\pi} G\left(\frac{R_0}{a}, \kappa, \ell_i, \ldots\right) I_P^2 \right\} \cdots\cdots (1)$$

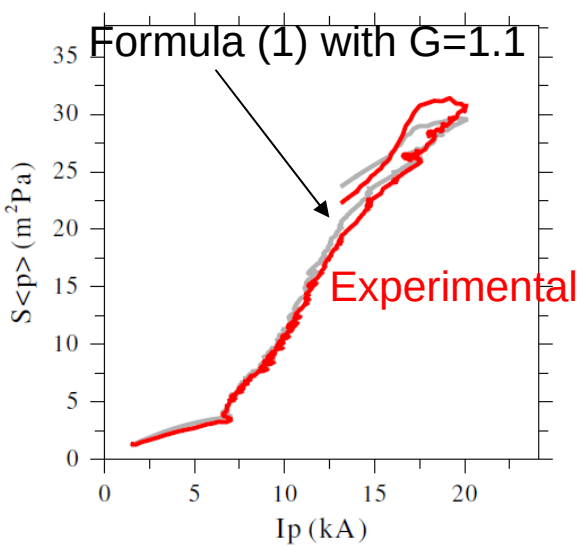
G is a fitting parameter that reflects geometrical parameters of current profile



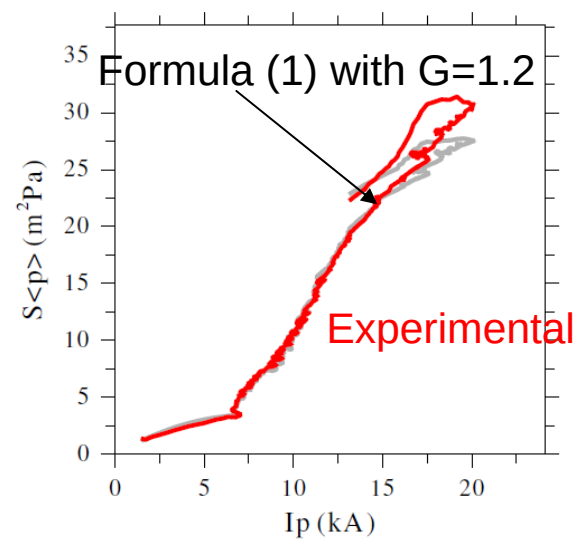
Formula (1) with G=1.0 and R_0=R_j and Bv at R=R_j



Formula (1) with G=1.1



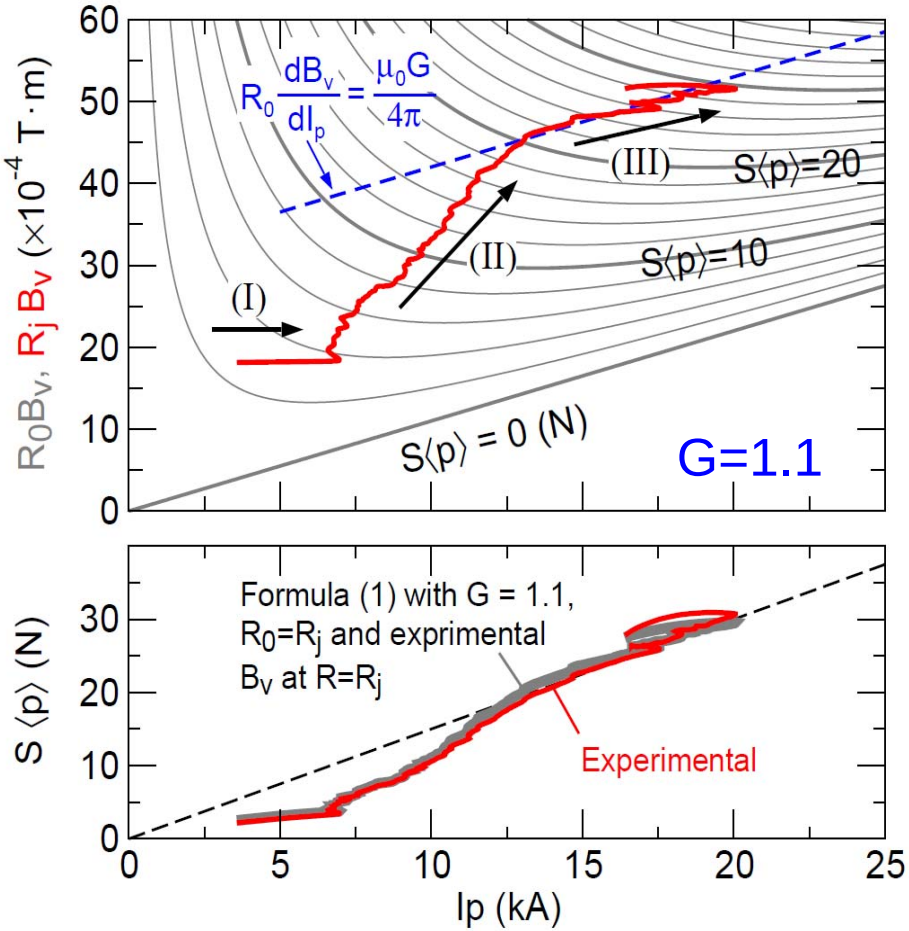
Formula (1) with G=1.2



The 20 kA discharge is compared with the formula on the contour map of S<P> on the Ip – R_0B_V space

$$S\langle P\rangle = \frac{1}{2}\left\{R_0 B_V I_P + \frac{\mu_0}{4\pi} G\left(\frac{R_0}{a}, \kappa, \ell_i, \ldots\right) I_P^2\right\} \cdots\cdots (1)$$

$$or \quad R_0 B_V = \frac{\mu_0}{4\pi} G\left(\frac{R_0}{a}, \kappa, \ell_i, \ldots\right) I_P + \frac{2S\langle P\rangle}{I_P}$$

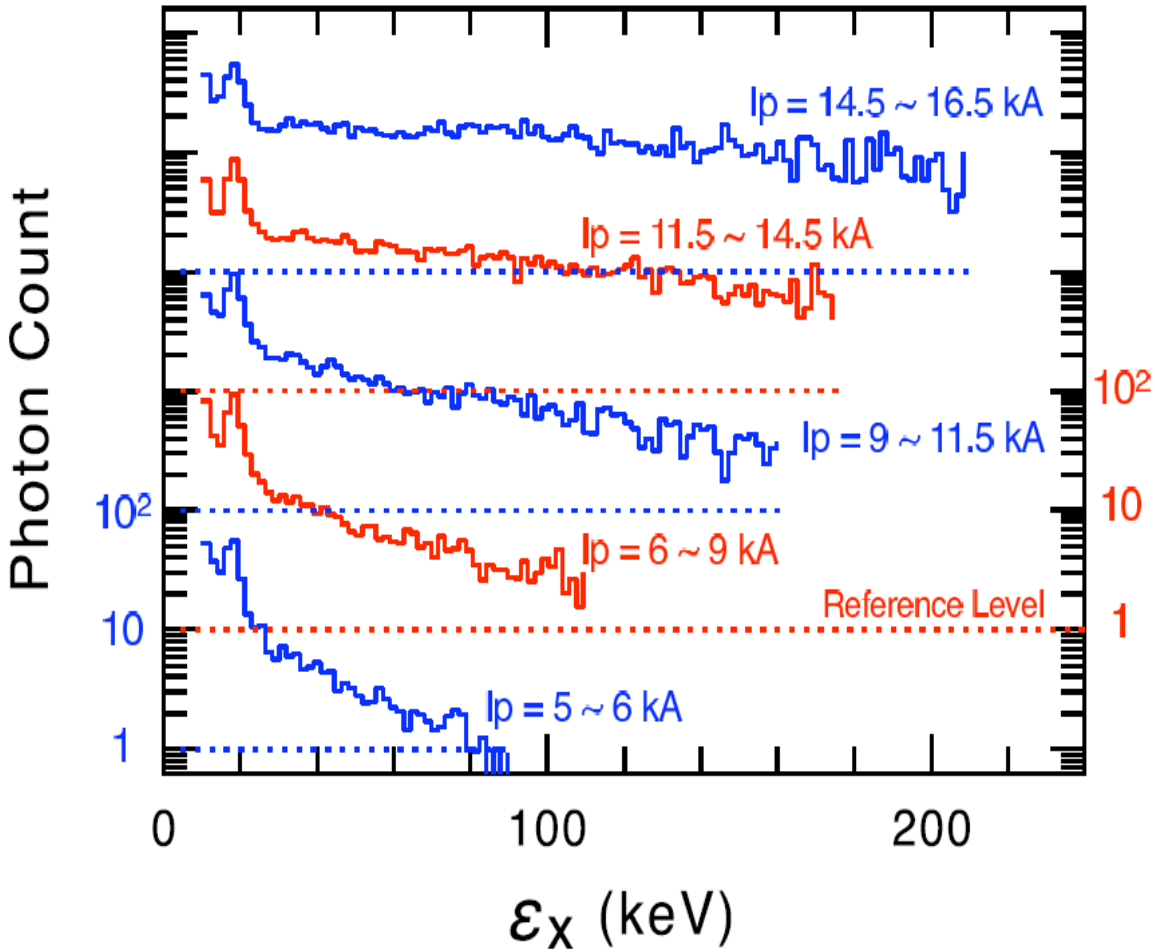


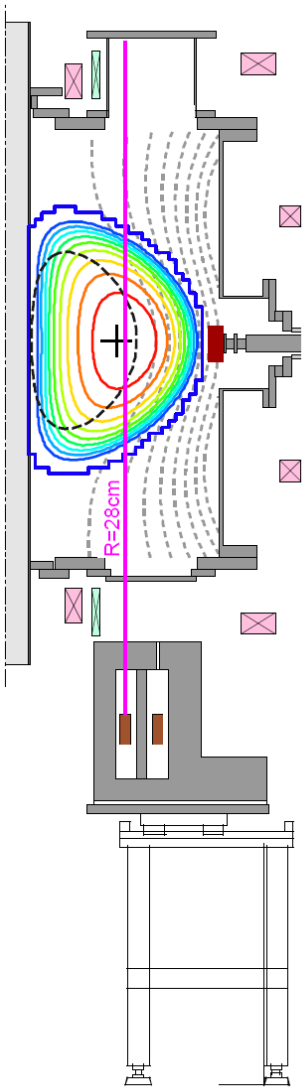
Stage II :
Pressure increases faster than Ip.
⇒ Energy range of current carrying fast electron tail expands with Ip.

Stage III:
Pressure increases in proportion to Ip.
⇒Energy range of fast electron tail becomes saturated.
Its population increases with Ip.

⇒dBv/dIp becomes low.
Induction from Bv field is low.
Induction from Ip-ramp is dominant and loop voltage becomes negative (VL~ - 0.1).

Hard X-ray energy range evolves as Ip ramps up.





Current outside LCFS is significant, implying that current carrying fats electron tail has anisotropic pressure.

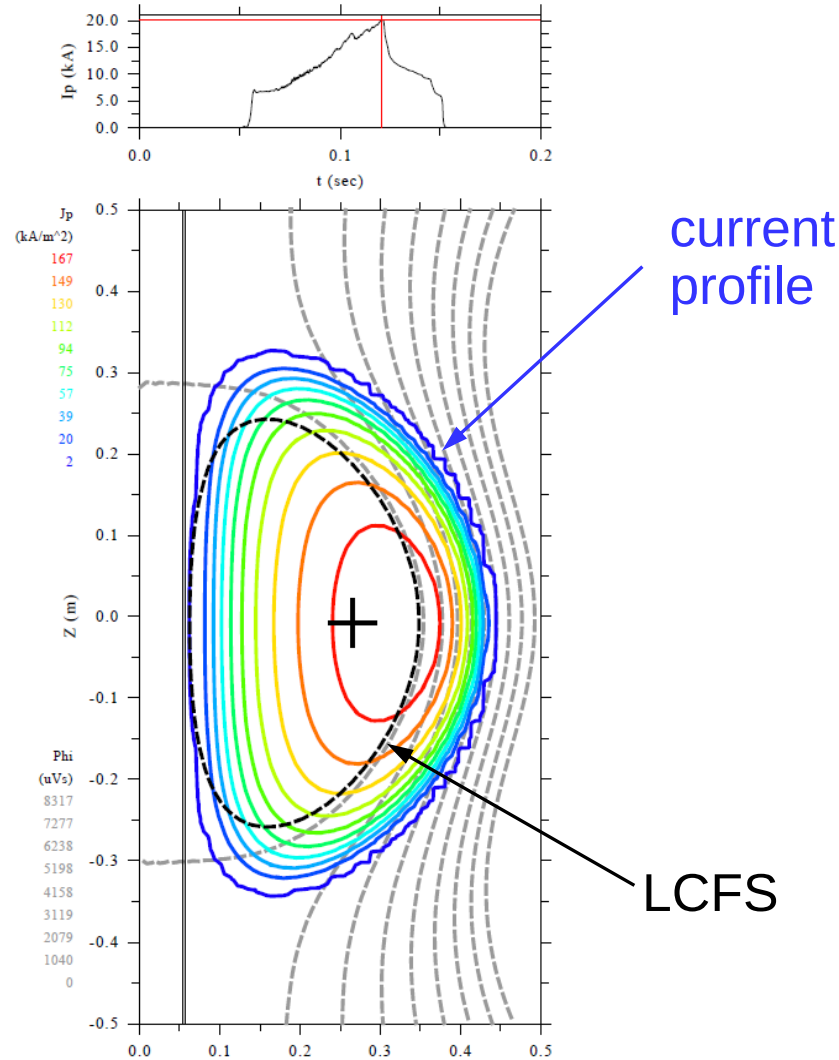
Current carrying electrons outside Last Closed Flux Surface

Field lines outside LCFS cross the vessel wall, on which plasma pressure is zero, implying some pressure gradient along the field lines.

In the case of isotropic pressure, there is no pressure gradient along the field line.

$$J \times B = \nabla P \quad \Rightarrow \quad B \cdot \nabla P = B \cdot J \times B = 0$$

We employ anisotropic pressure model.

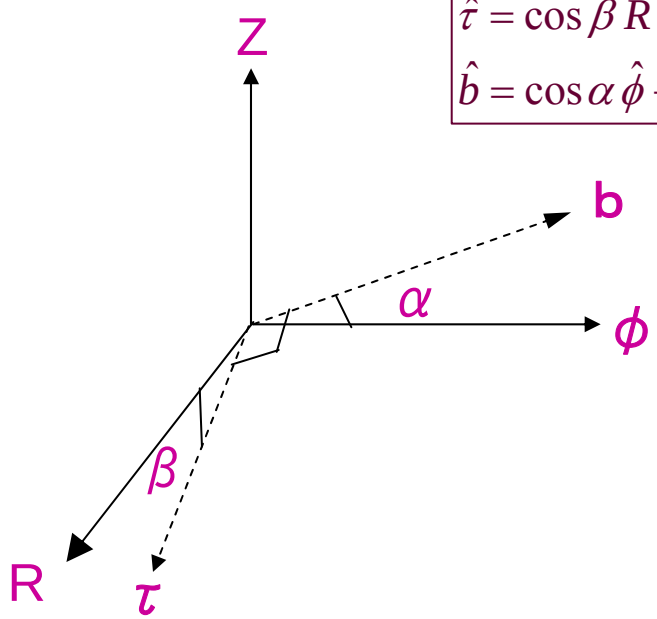


Local force balance (anisotropic pressure model)

$$\mathbf{j}\times\mathbf{B} = \nabla\cdot\overleftrightarrow{P} \qquad \overleftrightarrow{P} = p_\perp \overleftrightarrow{I} + (p_{//} - p_\perp)\hat{\mathbf{b}}\hat{\mathbf{b}} = \begin{pmatrix} p_\perp & 0 & 0 \\ 0 & p_\perp & 0 \\ 0 & 0 & p_{//} \end{pmatrix} \qquad \hat{\mathbf{b}} = \mathbf{B}/B$$

$$p_\perp = nm\langle \gamma v_\perp v_\perp/2\rangle \qquad p_{//} = nm\langle \gamma v_{//} v_{//}\rangle$$

$$\boxed{\begin{aligned} \hat{\tau} &= \cos\beta\,\hat{R} - \sin\beta\,\hat{Z} \\ \hat{b} &= \cos\alpha\,\hat{\phi} + \sin\alpha\cos\beta\,\hat{Z} + \sin\alpha\sin\beta\,\hat{R} \end{aligned}}$$



$$P_{RR} = p_\perp + (p_{//} - p_\perp)\sin^2\alpha\sin^2\beta$$

$$P_{ZZ} = p_\perp + (p_{//} - p_\perp)\sin^2\alpha\cos^2\beta$$

$$P_{\phi\phi} = p_{//} - (p_{//} - p_\perp)\sin^2\alpha$$

$$P_{R\phi} = P_{\phi R} = (p_{//} - p_\perp)\sin\alpha\cos\alpha\sin\beta$$

$$P_{\phi Z} = P_{Z\phi} = (p_{//} - p_\perp)\sin\alpha\cos\alpha\cos\beta$$

$$P_{ZR} = P_{RZ} = (p_{//} - p_\perp)\sin^2\alpha\sin\beta\cos\beta$$

Local force balance (anisotropic pressure model) (cont.)

$$R: \quad j_\phi B_Z - j_Z B_\phi = \frac{\partial P_{RR}}{\partial R} + \frac{\partial P_{ZR}}{\partial Z} + \frac{P_{RR} - P_{\phi\phi}}{R} \cong \frac{\partial p_\perp}{\partial R} + \frac{p_\perp - p_{//}}{R}$$

$$Z: \quad j_R B_\phi - j_\phi B_R = \frac{\partial P_{RZ}}{\partial R} + \frac{\partial P_{ZZ}}{\partial Z} + \frac{P_{RZ}}{R} \cong \frac{\partial p_\perp}{\partial Z}$$

together with the constraint $\nabla \bullet \mathbf{j} = \frac{1}{R}\frac{\partial}{\partial R}\left(R j_R\right) + \frac{\partial j_Z}{\partial Z} = 0$

$$\frac{\partial}{\partial Z}\left(p_\perp + p_{//}\right) = -2 j_\phi B_R - R\frac{\partial}{\partial Z}\left(j_\phi B_Z\right) - R\frac{\partial}{\partial R}\left(j_\phi B_R\right)$$

$$\phi: \quad j_Z B_R - j_R B_Z = \frac{\partial P_{R\phi}}{\partial R} + \frac{\partial P_{Z\phi}}{\partial Z} + \frac{P_{R\phi} + P_{\phi R}}{R}$$

$$\Rightarrow \quad \left(\hat{\mathbf{b}} \bullet \nabla\right) p_{//} - \left(p_\perp - p_{//}\right) F(R,Z) = 0$$

This equation is essentially identical to the force-balance equation along a field line.

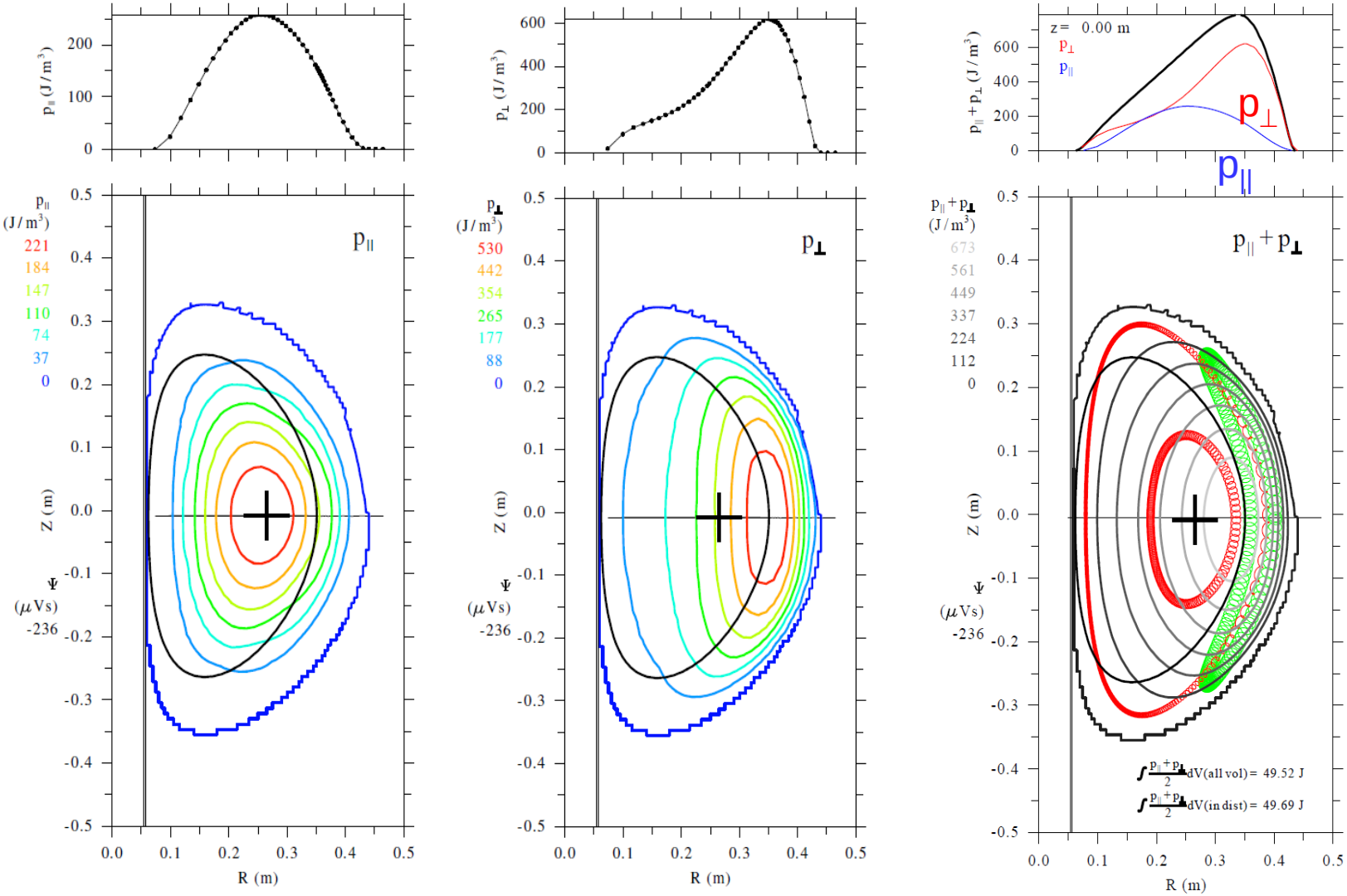
$$\left(\hat{\mathbf{b}} \bullet \nabla\right) p_{//} + \frac{\left(p_\perp - p_{//}\right)}{B}\left(\hat{\mathbf{b}} \bullet \nabla\right) B = 0$$

Equilibrium Pressure Profiles ($p_{||}$, $p_{\perp}$) deduced by anisotropic pressure model for the 20 kA plasma

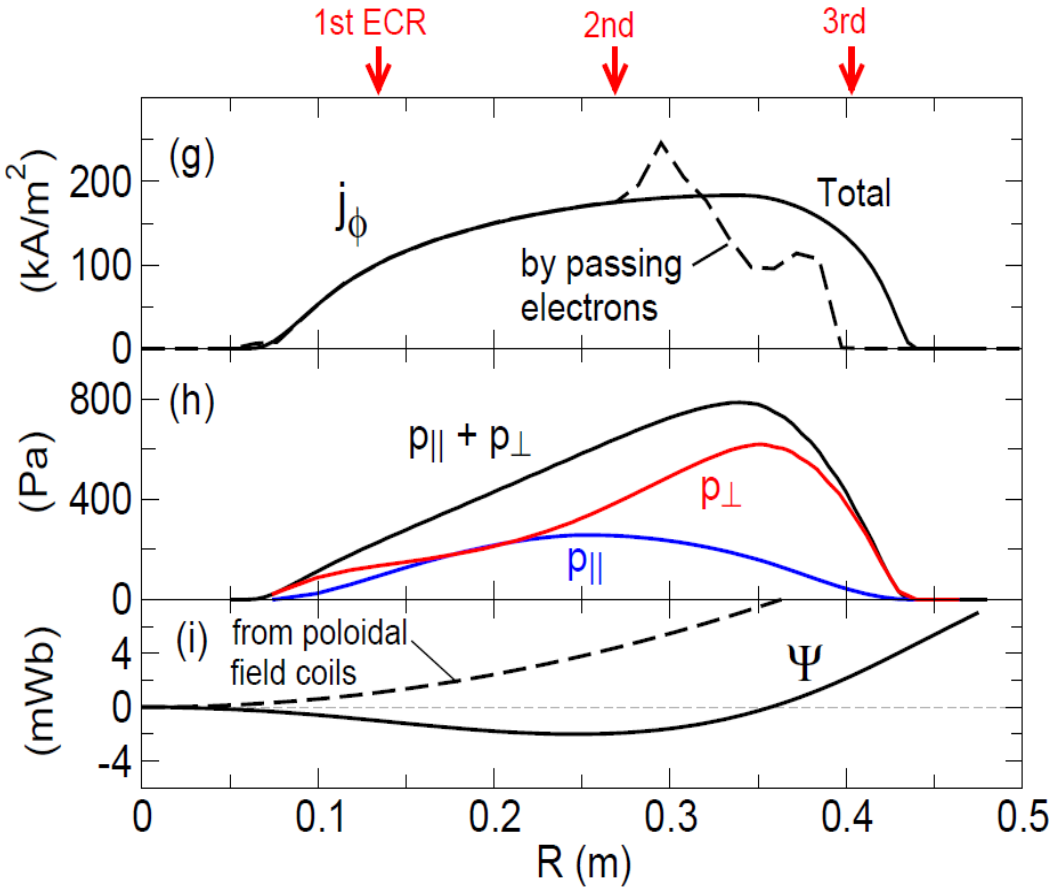
$p_{||}$ profile

$p_{\perp}$ profile

$p_{||}+p_{\perp}$ profile and electron orbits (100 keV)



Profiles on the mid-plane



Passing Electrons:

Density ~10^{10} cm^{-3}

Energy ~100 keV

Pitch angle ~60 degrees

These are estimated by assuming a monochromatic velocity distribution.

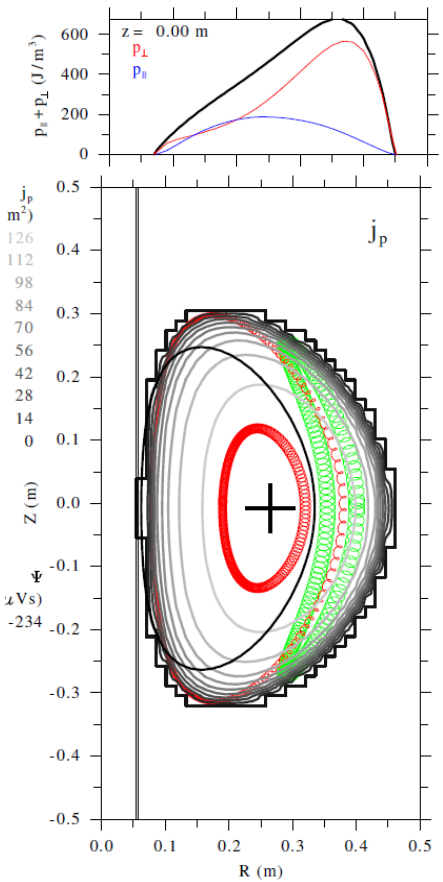
Current is carried mainly by passing electrons:

$I_{passing} / I_p \sim 75\%$

The rest current may be due to toroidal precession of trapped electrons.

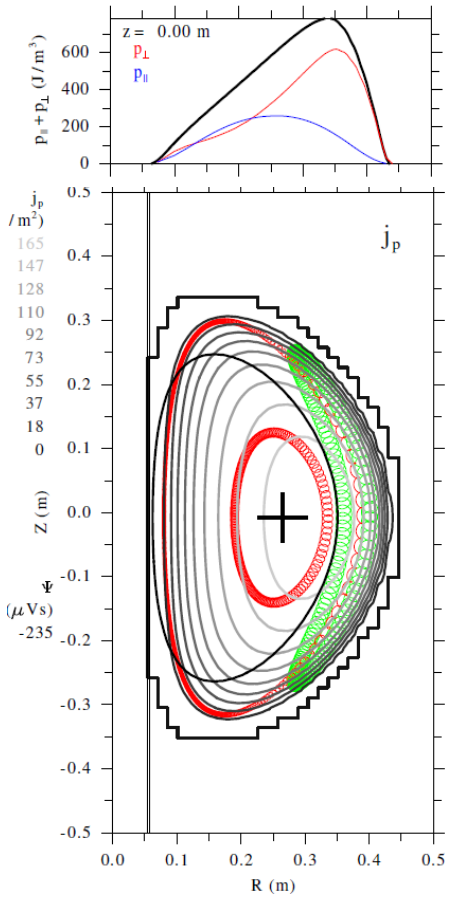
Other factors than the fitting error such as matching to the electron orbits and sign of pressure profile constrain the current profile.

Electron orbits do not match the current profile.



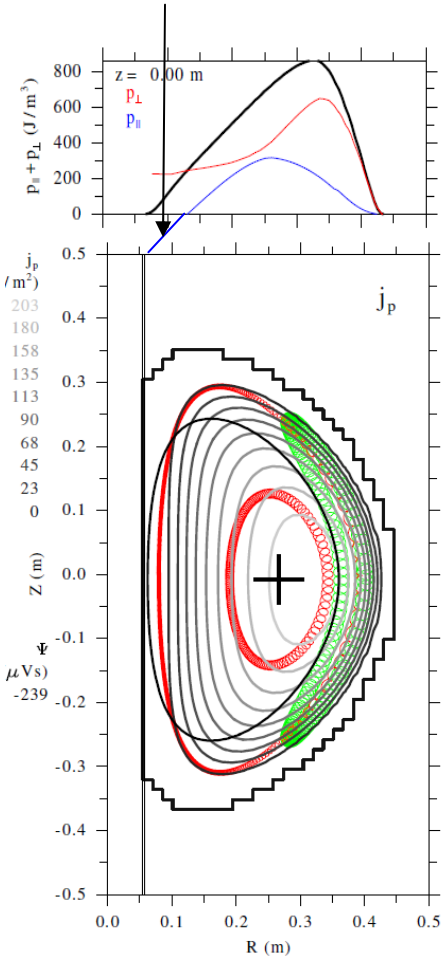
Current Peaking Index $\nu = 0.2$

Broad Profile



Current Peaking Index $\nu = 0.7$

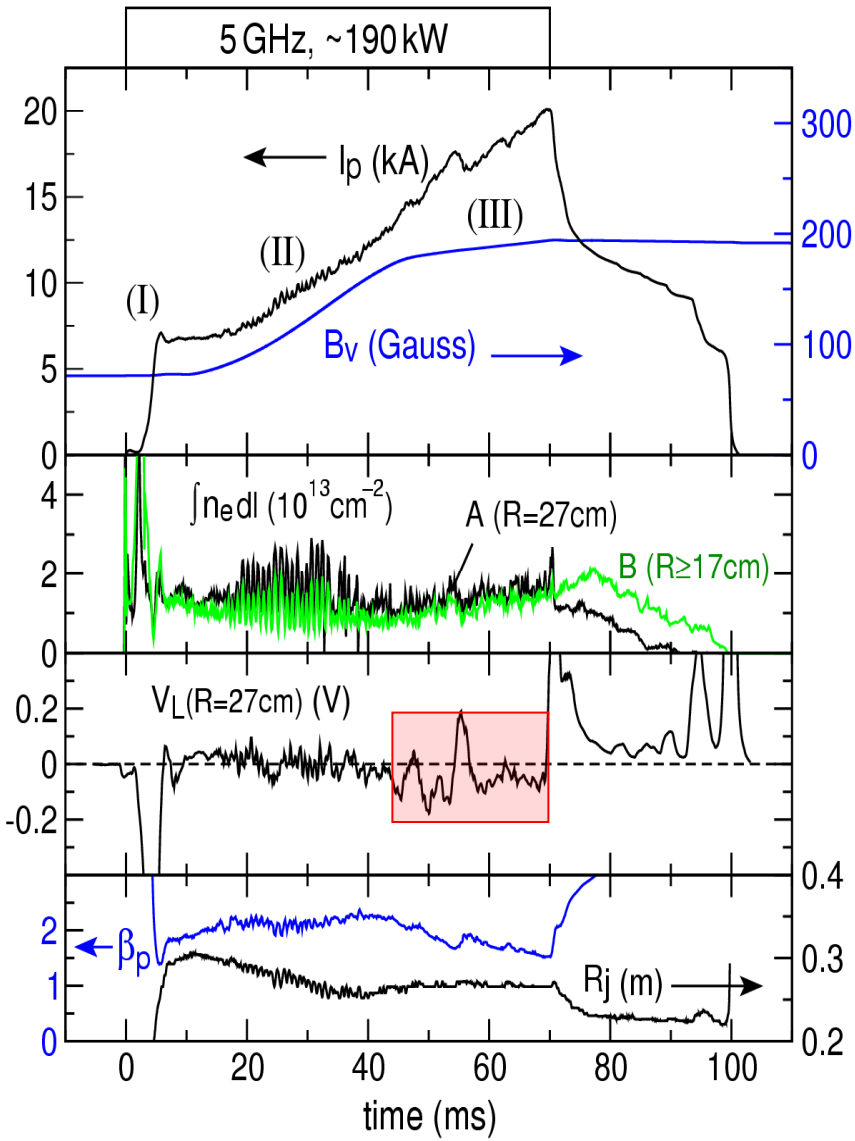
Negative P// appears.

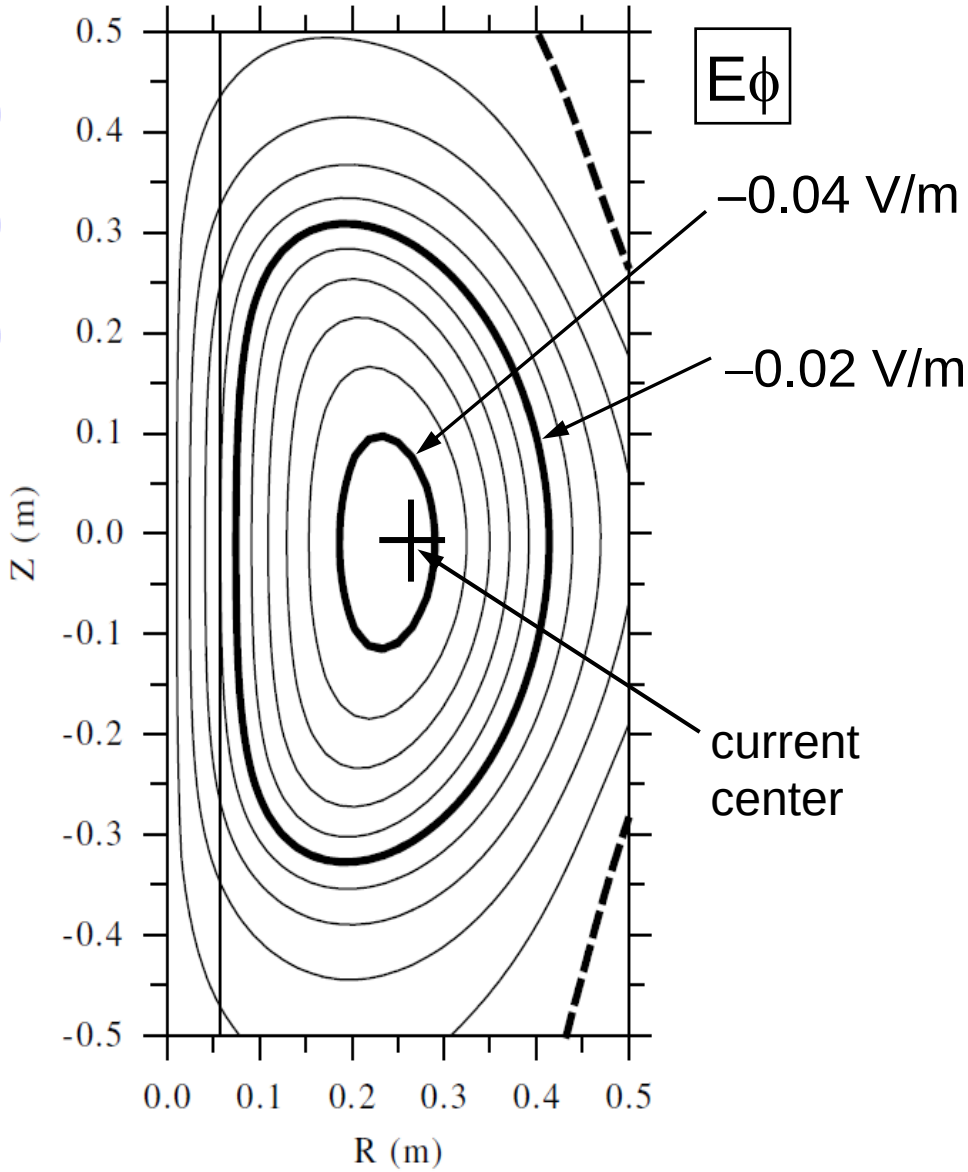


Current Peaking Index $\nu = 1.2$

Peaked Profile

Plasma current ramps up against the reverse electric field in the stage III





How do the tail electrons build up in velocity space ?

- Usually EC-resonance electrons are heated nearly perpendicularly to the magnetic field. Subsequent pitch-angle diffusion to the forward confinement region may build up the tail distribution.

However, the current ramp-up time (~ 50 ms) is two orders of magnitude shorter than the 90-degree scattering time for energetic tail electrons (~ 100keV) in the present discharge.

- Uni-directional fast electron tail far beyond the runaway critical energy (~ 4 keV) is generated against the reverse electric field of $E\phi$ = -0.02 V/m.

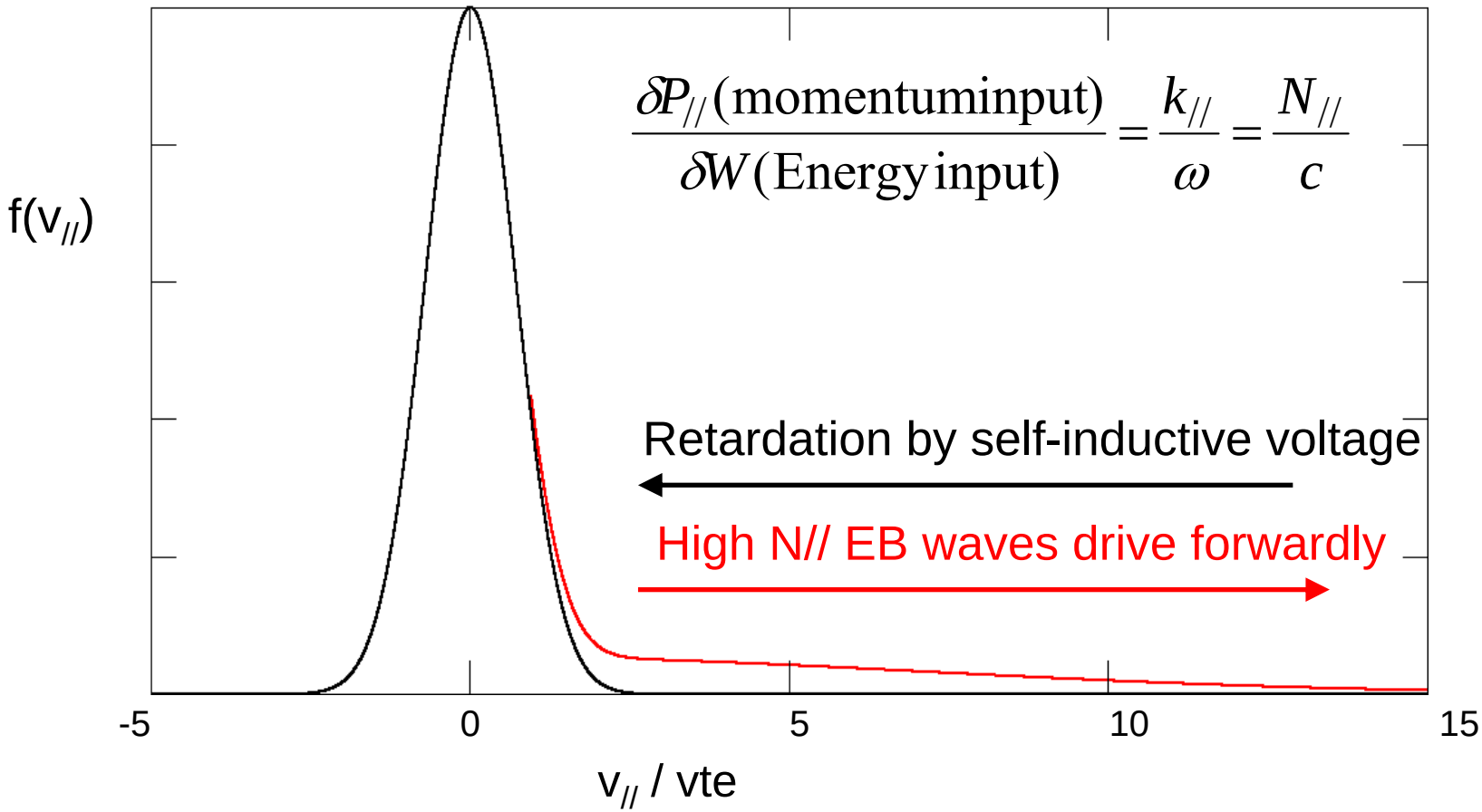
$$eE = \frac{m\mathrm{v_r}}{\tau_{col}} \qquad \mathrm{v}_r = \sqrt{\frac{e^3 \ln \Lambda}{4\pi\varepsilon_0^2 m}\frac{n}{E_\phi}} \approx 3.8\times10^7\, m/s \qquad \frac{m\mathrm{v}_r^2}{2} \approx 4\, keV$$

In the range of $v > v_r$, retardation by E field is larger than that by collision.

These results strongly suggest the presence of some direct forward force on the tail electrons.

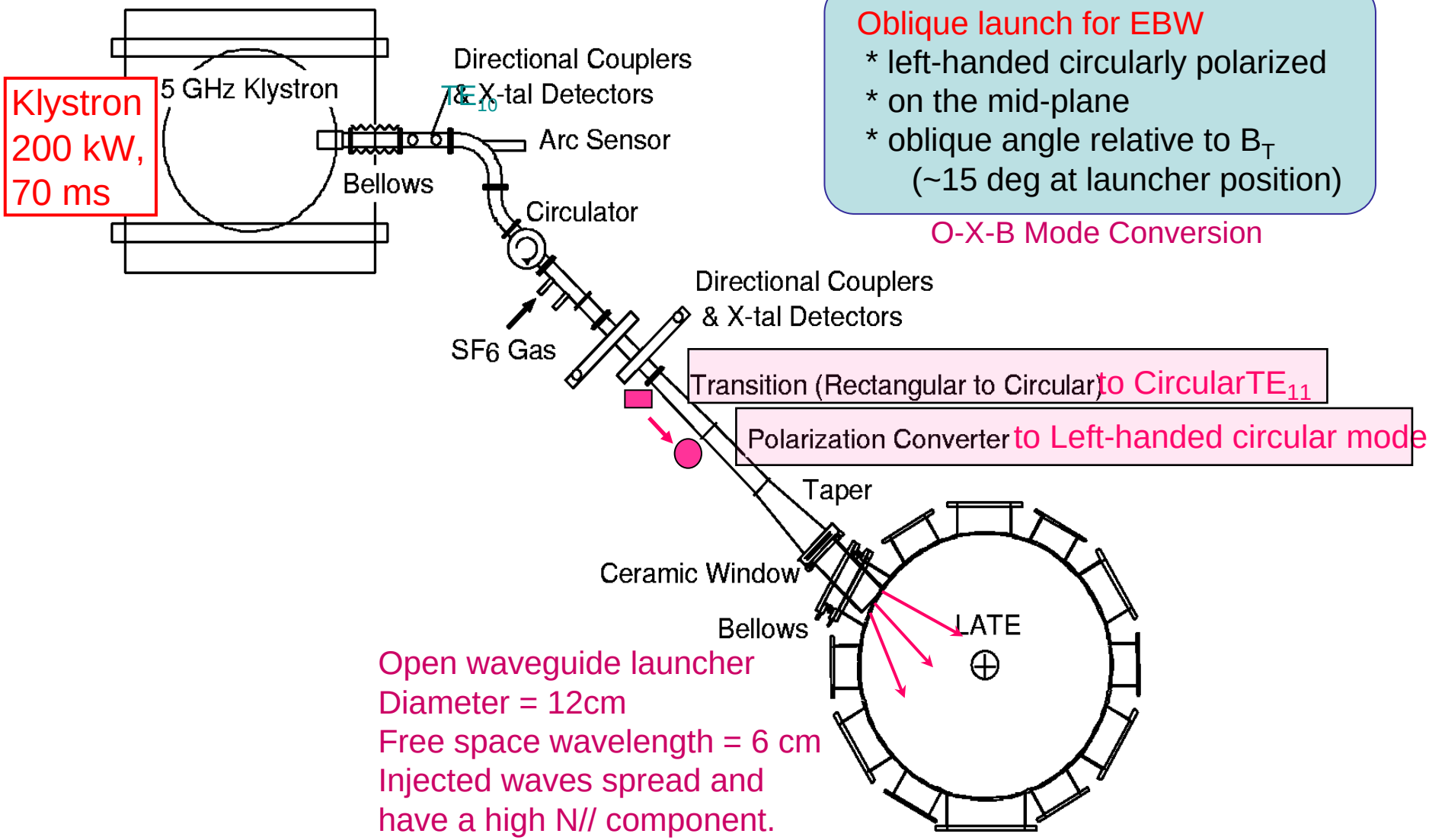
High N// EB waves can drive the tail electrons

Electron velocity distribution has a long current-carrying tail.



Plasma current may be ramped up by EBW-ECCD with high $N_{//}$.

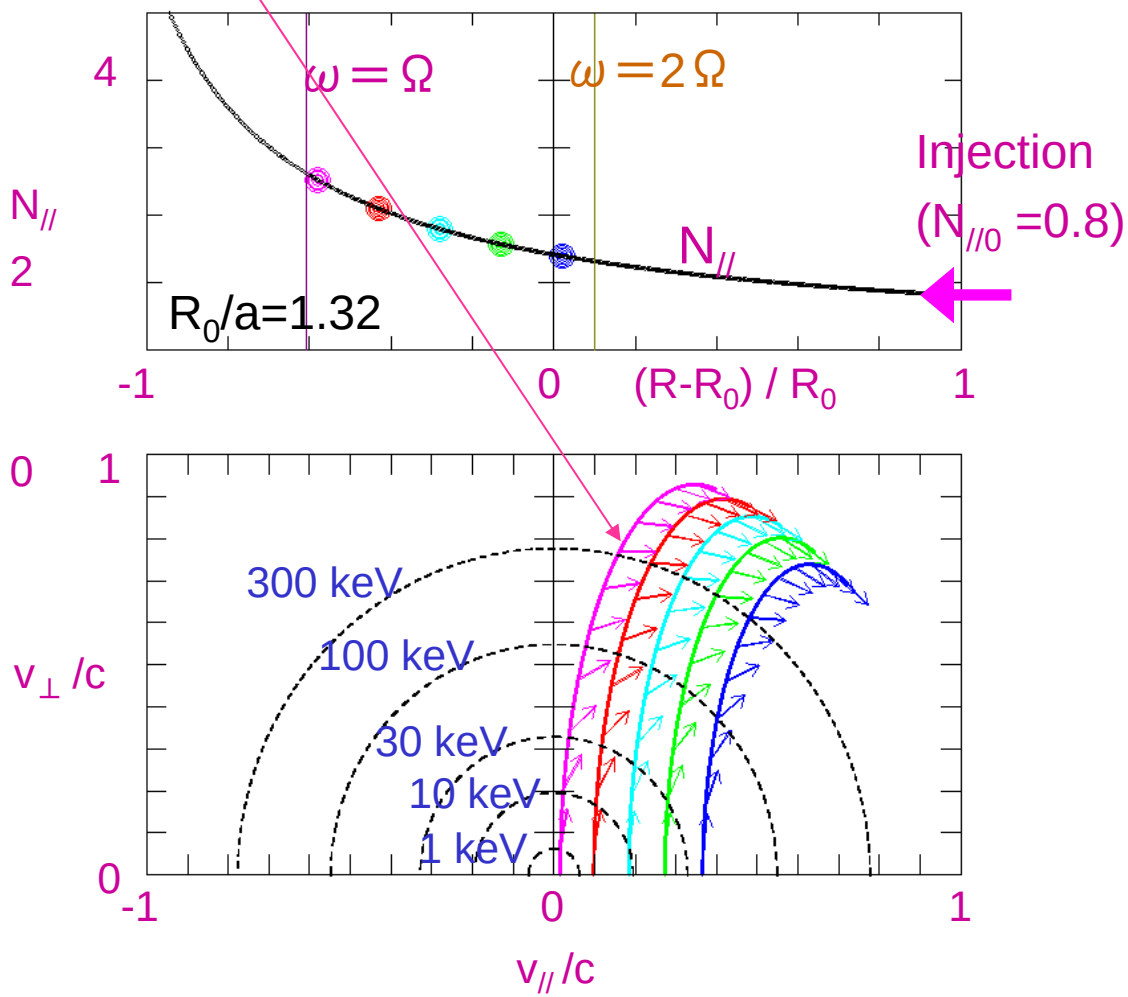
5 GHz Microwave System



In the stage III, high N// EB waves overdrive electrons from the thermal tail towards the energetic range well beyond the runaway velocity against the counter Electric force.

EC resonance ellipses for high $N_{//}$ Electron Bernstein Waves

(Arrows denote the direction of quasi-linear diffusion)



$$N_{//} = \frac{\mathbf{N} \bullet \mathbf{B}}{B} = \frac{N_\phi B_\phi + N_P B_P}{B}$$

$$\cong N_\phi + \frac{N_P B_P}{B}$$

$$N_\phi = \frac{N_{\phi 0} R_0}{R}$$

Toroidal wave length decreases as ∝R. Then N// increases as ∝1/R, significant in the low aspect ratio plasma.

Wave force that pushes resonance electrons parallel to the magnetic field is proportional to N//.

Electromagnetic waves (O and X modes) can not have high N//.

EB waves are an electrostatic mode and can have high N//.

Summary

Low aspect ratio tokamak plasma is produced by EC power and Bv control without induction from CS.

Discharge evolves in three stages:

I Closed surface is formed under steady Bv

II Ip ramps up with a ramp of Bv

III Bv-ramp is turned very slow. Ip still ramps up.

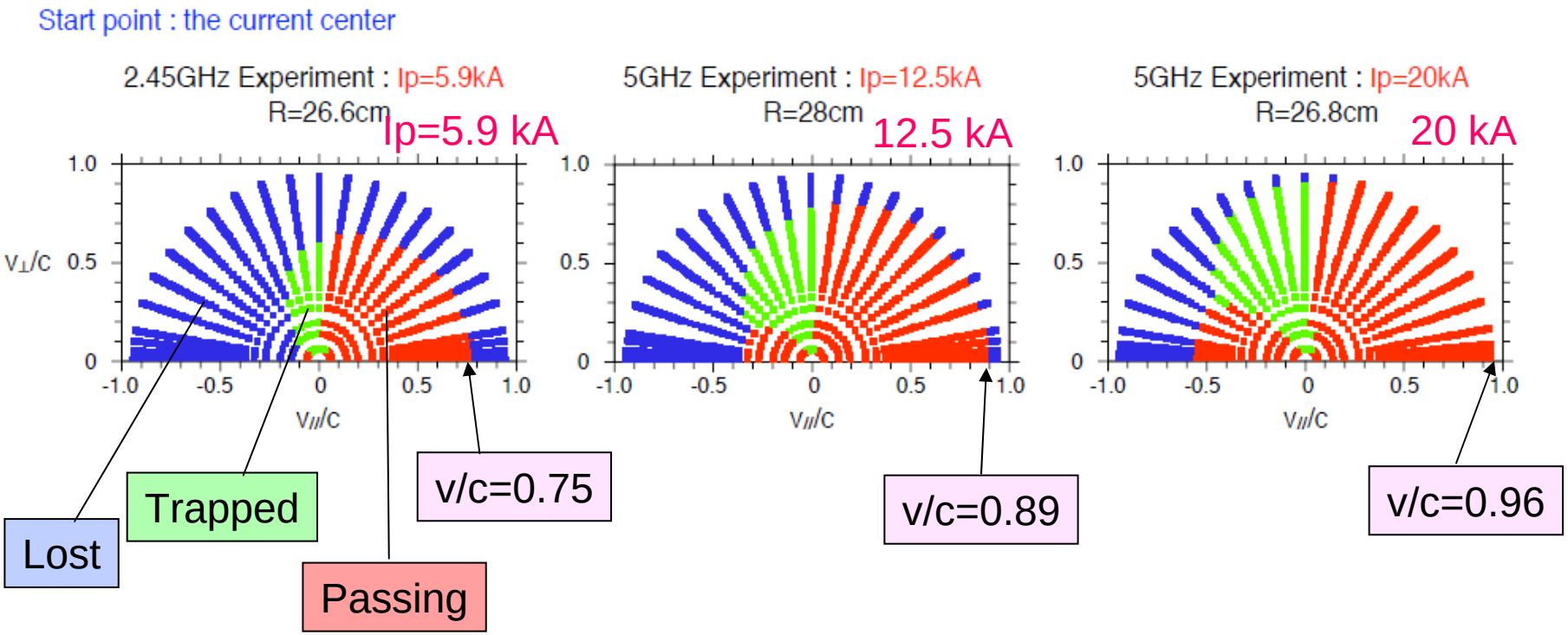
The last stage III is interesting. Equilibrium analyses, combined with the hard X-ray measurement and electron orbit analysis etc., elucidate the characteristics of the ramp-up plasma in this stage.

Current carrying fast electron tail (energy range up to ~ 200 keV) evolves against counter voltage from self induction.

This result strongly suggests forward driving force on the fast electrons by cyclotron absorption of high N// EB waves.

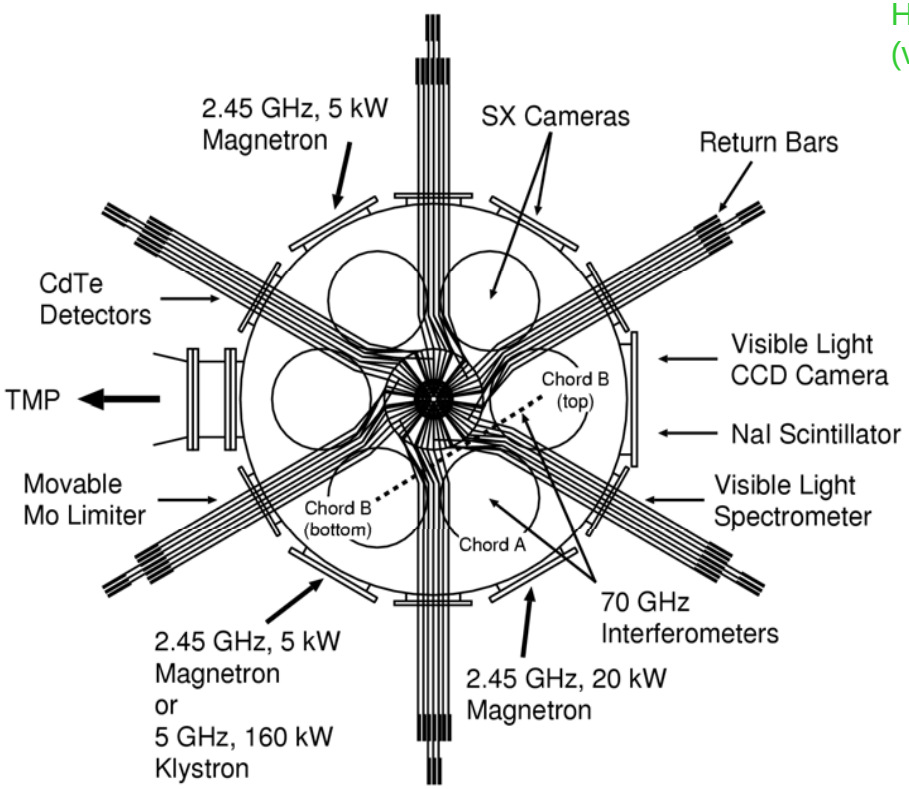
Back up

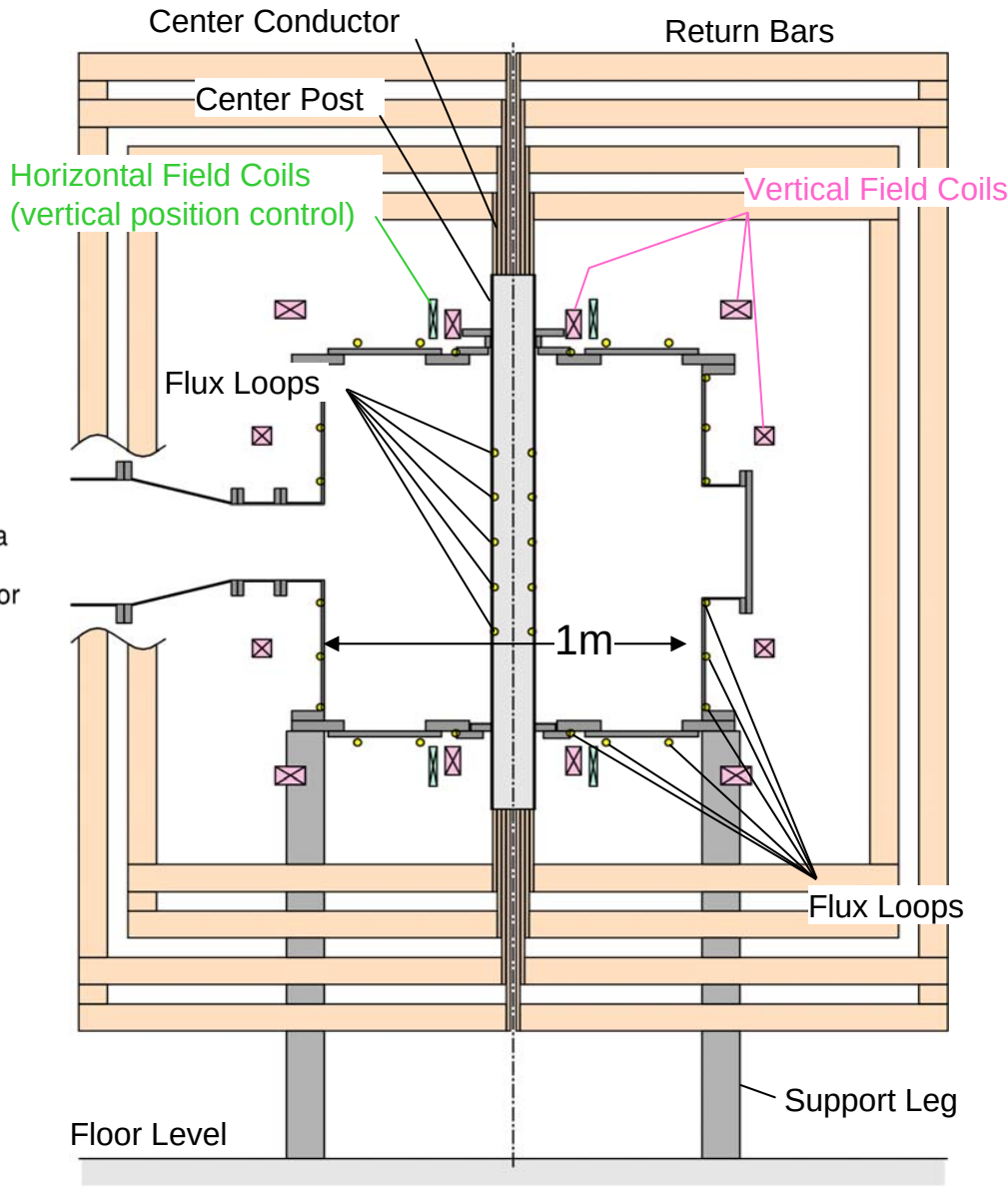
In the velocity space, orbit loss area decreases as Ip increases and almost disappears when Ip approaches ~20 kA.



Cutoff velocity v/c of passing electrons increases with Ip,
being consistent with that Cf and X-ray energy range increase with Ip.

LATE (Low Aspect ratio Torus Experiment) Device





■ No Central Solenoid

Parallel momentum of an electron evolves as

$$\frac{dP_{//}}{dt} = \frac{d\left(m\gamma \mathrm{v}_{//}\right)}{\mathrm{dt}} = q\left\{\mathbf{E}_1 + \mathbf{v}_0 \times \left(\mathbf{B}_1 + \mathbf{B}_0 \hat{z}\right)\right\} \cdot \hat{z}$$

$$= \frac{q}{\omega}\left\{\left(\omega - \mathbf{k}\cdot\mathbf{v}_0\right)E_{1//} + \left(\mathbf{v_0}\cdot\mathbf{E}_1\right)k_{//}\right\}\exp i\left(\mathbf{k}\cdot\mathbf{r}_0 - \omega t\right)$$

Here, $\mathbf{r}_0(t)$ *and* $\mathbf{v}_0(t)$ *are non perturbed orbit and velocity,*

and $\omega\mathbf{B}_1 = k \times \mathbf{E}_1$ *is used.* $\mathbf{E}_1$ *and* $\mathbf{B}_1$ *are wave fields.*

Notice that

$$i\left(\omega - \mathbf{k}\cdot\mathbf{v}_0(t)\right)E_{1//}\exp i\left(\mathbf{k}\cdot\mathbf{r}_0(t) - \omega t\right) = -\frac{d}{dt}\left\{E_{1//}\exp i\left(\mathbf{k}\cdot\mathbf{r}_0(t) - \omega t\right)\right\}.$$

After time average,

$$\left\langle\frac{dP_{//}}{dt}\right\rangle = \frac{k_{//}}{\omega}\left\langle q\mathbf{v_0}\cdot\mathbf{E}_1\right\rangle = \frac{k_{//}}{\omega}\left\langle\frac{dW}{dt}\right\rangle = \frac{N_{//}}{c}\left\langle\frac{dW}{dt}\right\rangle; \quad W \text{ is the kinetic energy.}$$